The linear stability of plane creeping Poiseuille and Couette flow of Burgers fluid

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Summary The linear stability of plane creeping Poiseuille and Couette flow of Burgers fluid are studied. Unstable modes are detected with some special value of parameters, which is different from the previous results of UCM and Oldroy-B fluids. The wave speed of critical stable wave contains one component, which is caused by elasticity and dominant at low Reynolds numbers. Unstable modes can still be found at high wave numbers. The growth rate of Couette flow increases monotonically with the wave number. The modules and phases of critical stable modes show symmetry in wall-normal direction for both flows. Energy analysis shows the maintenances of perturbation energy for Poiseuille and for Couette flow are different. The former comes from the power made by different components of the perturbation stress while the latter comes from of the anti-symmetry of perturbation stress.

INTRODUCTION

The linear stability of plane Poiseuille and Couette flow for viscoelastic fluids has been studied for a long-time [1-3]. Although elasticity can destabilize the flow significantly, this destabilizing effect cannot lead to instability for UCM and Oldroy-B fluids. Therefore, the melt fracture which exists at very low Reynolds number flow for many viscoelastic fluids cannot be explained by the linear instability theory with UCM and Oldroyd-B models. However unstable modes have been found for plane Poiseuille flow of Burgers fluid at very low Reynolds numbers [4].
The Burgers fluid is presented by Burgers whose mechanical analogy is a Maxwell model and a Vogit model connected in series [5]. The UCM and Oldroyd-B fluids can be treated as the special cases of it. Recent studies show that Burgers fluid can model many viscoelastic materials, such as earth mantle, asphalt mixtures, polymers et.al.
This paper studies the linear stability of plane creeping Poiseuille and Couette flow of Burgers fluid. Unstable modes are found in some parameters. We study the wave speed, the modes, the growth rate and the maintenance of energy for the perturbation wave.

SOLUTION OF THE BASIC FLOW AND EQUATIONS FOR THE PERTURBATION FLOW

The dimensionless constitutive equation of Burgers fluid can be derived as

$$\mathbf{T}+\lambda\frac{\delta\mathbf{T}}{\delta t}+\beta\frac{\delta^2\mathbf{T}}{\delta t^2}=\left(1+\eta\frac{\delta}{\delta t}\right)\mathbf{A}\,, \tag{1}$$

where $\mathbf{T}$ is the stress tensor, $\mathbf{A}$ is the strain-rate tensor, λ,β,η are three dimensionless parameters of Burgers fluid.
Let x direction be the streamwise direction, and y direction be the wall-normal direction. The characteristic length L is chosen as half the width of the slot, so the dimensionless values $y=\pm 1$ stand for the two solid boundaries. The characteristic velocity U is chosen as the fastest velocity of the basic flow. Suppose the perturbation flow has the form below

$$\mathbf{u}=\mathbf{u_0}+\mathbf{u}_1(y)\exp(i\alpha x+\sigma t),\quad \mathbf{u}_1(y)=(u,v),\quad \sigma=\mp \mathrm{i}\alpha C\,, \tag{2}$$

where the subscript 0 stands for the basic flow, α is the dimensionless wave number. Chebyshev polynomials are used to compute the eigenvalue σ, C is the wave speed. As Couette flow is anti-symmetrical with respect to y-axis, we focus on the wave that has $\sigma=-\mathrm{i}\alpha C$ below.

NUMERICAL RESULTS

By computation, we find unstable modes for both flows. For example the growth rate of Couette flow $\mathrm{Re}(\sigma)>0$ when Reynolds number $R<10^{-3}$ for $\lambda=3$, $\beta=1.4$, $\eta=1.4$, $\alpha=2.6441$. For both flows, the wave speed of critical stable wave contains one component, which is inversely proportional to the one-half power of the Reynolds number and dominant in the creeping region. It is caused by elasticity and keeps constant when the Reynolds number changes.
Then we study the perturbation modes in critical stable case. The perturbation velocities u,v are both complex values. Write them in the form below

$$u(y)=|u(y)|\exp(i\theta_1(y)),\quad v(y)=|v(y)|\exp(i\theta_2(y))\,, \tag{3}$$

The modules of perturbation mode are symmetric with respect to y-axis for both flows. Fig. 1 shows that of Couette flow for $R=10^{-3}$ **.**Other parameters are the same as above. The phases of perturbation mode for Poiseuille flow are symmetric with respect to y-axis for v but anti-symmetric for u **.** For Couette flow, the phases are anti-symmetric when they minus a constant, which is shown in Fig. 2.

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Unstable modes can still be found at high wave numbers for both flows, which is different of the case for Newtonian fluid. The growth rate of Couette flow increases monotonically with the wave number and finally reaches a stabilized value.

The cause of the instability is studied by energy analysis. It shows the maintenances of perturbation energy for Poiseuille and Couette flow are different. The former comes from the power made by different components of the perturbation stress while the latter comes from of the anti-symmetry of perturbation stress. The latter is shown in Fig. 3 where E_{xx}, E_{yy} are the power made by the perturbation normal stress in x-axis and y-axis, E_{xy} are the power made by the perturbation shear stress. The parameters are the same as those in Fig. 1. The power made by the perturbation stress is basically anti-symmetric with respect to y-axis. We also find that the anti-symmetry will get better when the Reynolds number reduces.

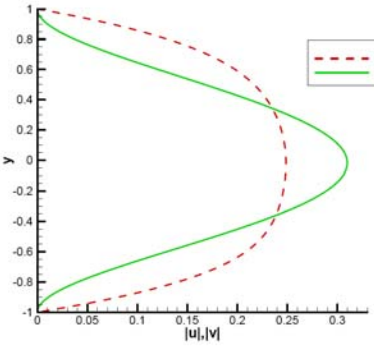

Fig. 1 The modules of perturbation

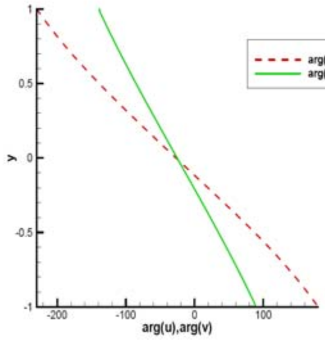

Fig. 2 The phases of perturbation

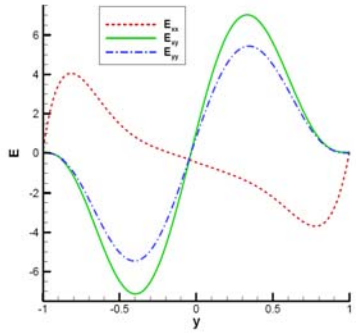

Fig. 3 The power made by the perturbation stress

ANALYSIS IN LOW REYNOLDS NUMBER LIMIT

From the previous calculation, we find $|\sigma| \to \infty$ when $R \to 0$. Using this property, the governing equations of Couette flow can be simplified and has analytic solution below

$$v(y) = \sum_{j=1}^{4} A_j \exp(k_j \cdot i\alpha y), \tag{4}$$

where $k_j\,(j = 1 \sim 4)$ are the roots of the equations below

$$bk^4 + k^3 + (2b + a)k^2 + 2k + (b + a) = M(k^2 + 1), \quad a = 2(\lambda - \eta), \quad b = \eta / \beta, \quad M = C^2 R, \tag{5}$$

and $A_j\,(j = 1 \sim 4)$ are constants whose relations can be determined by the boundary conditions below

$$v'(\pm 1) = v(\pm 1) = 0, \tag{6}$$

Thus it is easy to verify that the modules of v are symmetric with respect to y-axis while the phases are anti-symmetric when they minus a constant. These properties are true for u and perturbation stress. Therefore, it can be deduced that E_{xx}, E_{xy}, E_{yy} are all are symmetric with respect to y-axis. This can explain the symmetry in Fig.1-3.

CONCLUSIONS

In this paper, we study the linear stability of plane creeping Poiseuille flow and Couette flow of Burgers fluid. Different from UCM and Oldroy-B fluids, unstable modes are detected with some special value of parameters. The wave speed of critical stable wave contains one component, which is inversely proportional to the one-half power of the Reynolds number and dominant in creeping region. It is caused by elasticity and keeps constant when the Reynolds number changes. Unstable modes can still be found at high wave numbers for both flows. The growth rate of Couette flow increases monotonically with the wave number and finally reaches a stabilized value. For the critical stable modes, the modules and phases show symmetry in wall-normal direction for both flows. Energy analysis shows the maintenance of perturbation energy for Poiseuille and Couette flow are different. The former comes from the power made by different components of the perturbation stress while the latter comes from of the anti-symmetry of perturbation stress. We also analyze the perturbation in low Reynolds number limit for Couette flow and explain the symmetry in the figures.

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