A Numerical Study of the MPS Method for Simulating Non-Newtonian Free Surface Flows

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Summary The Moving Particle Semi-implicit method (MPS) is a Lagrangian particle method suitable for surface flows. This paper couples the non-Newtonian model with the MPS method by treating the non-Newtonian fluid as Newtonian fluid with variable viscosity. In theory, all non-Newtonian fluids have a constitutive equation as $\mu=f(|\gamma|)$,which can be treated by this method. In this paper, the formulation is demonstrated using the Casson and Cross rheological models. A 2D mudflow dam-break problem is investigated by present method. The computations are in good agreement with available experimental data and previous simulations. The results show the Casson and Cross flow propagates more slowly than the Newtonian flow because of their viscous nature.
Key words: MPS; free surface; Casson model; Cross model; dam-break

INTRODUCTION

Free surface flow is a significant environmental and industrial phenomenon which has been studied for many years in computational fluid dynamics. Recently, a Lagrangian particle method called moving particle semi-implicit method (MPS) drew attention. The MPS method was proposed by Koshizuka et al. [1] in 1996. Compared with traditional grid methods, no extra equations and special treatments for interface capturing and reconstruction are needed. Furthermore, fully Lagrangian treatment avoids numerical diffusion. The motivation of this paper is to extend the MPS method for the simulation of some typical non-Newtonian fluids.

NUMERICAL ALGORITHM

Governing equations

For incompressible Newtonian flow, the Lagrangian governing equations can be written as:

$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\nabla\cdot\boldsymbol{u} = 0 \tag{1}$$

$$\rho\frac{\partial\boldsymbol{u}}{\partial t} = -\nabla p + \mu\nabla^2\boldsymbol{u} + \boldsymbol{F} \tag{2}$$

where $\boldsymbol{u}$ denotes particle velocity; ρ denotes fluid particle density; t denotes time; $\boldsymbol{F}$ denotes mass force which is usually gravity and μ denotes dynamic viscosity. In the MPS method, the flow field is discretized using finite particles. Each particle interacts with its neighboring particles by a weighting function. The commonly used weighting function, gradient operator, Laplacian operator and predictor-corrector method can be seen in the reference [1].

Non-Newtonian model

For a Newtonian fluid, the dynamic viscosity μ is constant. To the contrary, for the non-Newtonian fluid, μ is a function of shear rate γ. The Casson model and Cross model will be taken as examples to explain how to couple the non-Newtonian model with the MPS method.

The Casson model was proposed by Casson [2] in 1959. For making the function continuous, the Papanastasiou approximate model [3] is adopted as

$$\mu\left(|\gamma|^*\right) = \mu_\infty\left[1+\sqrt{\frac{Bn}{|\gamma|^*}}\left(1-e^{-\sqrt{m'\cdot|\gamma|^*}}\right)\right]^2 \tag{3}$$

Let $Bn=\tau_y l/\mu_\infty U_\infty$ and $m'=mU_\infty/l$, where l is characteristic length of flow field, U_∞ is maximum velocity, τ denotes shear stress, τ_y denotes yield stress, μ_∞ is the limiting viscosity and m is a constant model parameter. $|\gamma|^*$ is defined as:

$$|\gamma|^* = \frac{l}{U_\infty}\sqrt{2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2} \tag{4}$$

After aforementioned treatment, γ can be obtained by the velocity field of previous time step and then dynamic viscosity μ of current time step is obtained by Eq (3). Therefore, Eq (2) is still suitable for the non-Newtonian fluid when we treat μ as a variable. Obviously, the non-Newtonian fluid can be approached as Newtonian fluid with variable viscosity as long as its dynamic viscosity μ can be written as a function of $\mu=f(|\gamma|)$.

Similarly, according to Shao & Lo [4], the constitutive relationship of Cross model can be expressed as:

$$\mu = \left(1000\mu_B + \frac{1000\mu_B{}^2}{\tau_B}|\gamma|\right)\Bigg/\left(1+\frac{1000\mu_B}{\tau_B}|\gamma|\right) \tag{5}$$

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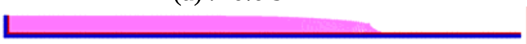
where μ_B is Bingham yield viscosity and τ_B is Bingham yield stress. Similar to the Casson model, μ is a function of γ, so we can adopt the same processing method used in the Casson model to Cross model.

NUMERICAL TESTS

A 2D mudflow dam-break problem is investigated by present method. The initial mixture length and height is 2.0 m and 0.1 m respectively and the channel bed slope S_0=0.1% is considered. The mud has density of 1200 kg/m^3, Bingham yield stress of 25.0 Pa and viscosity of 0.07 Ns/m^2,, which can be treated as the Cross model. There are 2000 fluid particles and 1256 boundary particles with initial spacing of 0.01 m. Particle configurations at t=0.0, 0.1, 0.3 and 0.6s are shown as Fig. 1. It is worth to mention that r_e is parallel set to 2.1 and 3.1.

(a) t=0.0 s

(b) t=0.1s

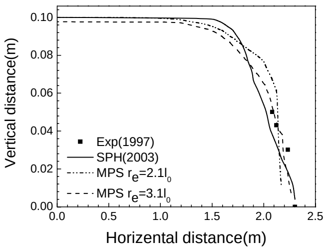


(c) t=0.3s

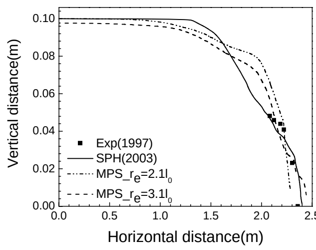
(d) t=0.6s

Fig. 1 Particle configurations at t=0.0, 0.1, 0.3 and 0.6s

We compare the surface profiles of mudflow at t=0.37 and 0.6 s as shown in Fig. 2. The result of r_e=2.1 and 3.1 are both acceptable comparing with the SPH simulation and the experimental data [5]. By contrast, when r_e=3.1, the result of front body of dam-break anastomoses the experimental data better than that of r_e=2.1 and the SPH simulation. It is reasonable that when the effective influencing radius increases from 2.1 to 3.1, there are more particles involved in computation, which improves the calculation precision. Newtonian and Casson fluids with the same flow parameters as the Cross model are also considered in Fig. 3. Compared with the Newtonian flow, the Casson and Cross flow propagate more slowly because of their viscous nature.

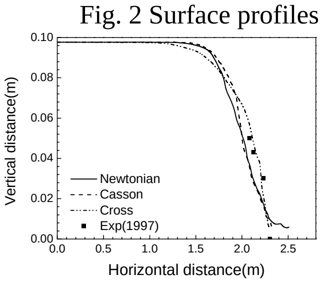


(a) t=0.37 s

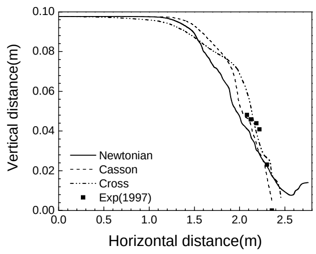


(b) t=0.6 s

Fig. 2 Surface profiles at (a) t=0.37 s and (b) t=0.6s



(a) t=0.37 s



(b) t=0.6 s

Fig. 3 the surface profiles of Cross model, Casson model and Newtonian model at (a) t=0.37 s and (b) t=0.6s

CONCLUSION

This paper presents the MPS method to simulate non-Newtonian flows with free surfaces by treating the non-Newtonian fluid as Newtonian fluid with variable viscosity. This method is validated as long as the dynamic viscosity μ of the Newtonian fluid can be written as a function of $\mu=f(|\gamma|)$. The Casson model and the Cross model are applied for an example to derive the constitutive equations which can be coupled with the MPS method and use them to simulate a mudflow dam-break problem. The results of the MPS method are in better agreement with the experimental data then SPH computation. It shows the Casson and Cross flow propagates more slowly than the Newtonian flow because of their viscous nature.

References

[1] Koshizuka S., Oka Y.: Moving-particle semi-implicit method for fragmentation of incompressible fluid. *NUCL SCI ENG*, 1996, 123(3): 421–434.

[2] Casson N.: Rheology of disperse systems. London: CC Mill - Pergamon Press, 1959.

[3] Papanastasiou T.C.: Flows of materials with yield. Journal of Rheology, 1987, 31(5): 385-404.

[4] S.D. Shao, E. Lo.: Incompressible SPH method for simulating Newtonian and non-Newtonian flows with a free surface. *ADV WATER RESOUR*, 2003, 26(7): 787-800.

[5] D. Komatina, M. Jovanovic.: Experimental study of steady and unsteady free surface flows with water-clay mixtures. *J HYDRAUL RES*, 1997, 35(5): 579-590.