ONSET OF THERMAL CONVECTION IN A VISCOELASTIC FLUID-SATURATED POROUS MEDIUM BETWEEN COAXIAL ROTATING CYLINDERS

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Summary Linear stability analysis of thermal convection in a viscoelastic fluid-saturated rotating porous medium between two coaxial cylinders is studied. The effects of rotation due to Coriolis force is considered. The three dimensional marginal curve together with corresponding mode maps is given. The effects of rotation and viscoelastic parameters on the oscillatory convection are analyzed.

INTRODUCTION

The classical Rayleigh-Benárd convection(RBC) in porous media has received considerable attention, and continues to be a topic of intense research. A particularly interesting variation of RBC in porous media is the case where the sample is rotated about a vertical axis with a uniform angular speed. That system is relevant to many engineering applications, such as extraction processes in food or chemical industry, and rotating machinery. Vadasz [1] studied the effect of rotation due to Coriolis force in a rotating porous layer for a Newtonian fluid. On the other hand, it has been widely understood that onset of thermal convection for viscoelastic fluids is different from that for Newtonian fluids. Loss of stability for a viscoelastic fluid has two different mechanisms [2]: the principle of exchange of stabilities and overstability, but only the former holds for a Newtonian fluid. The effects of viscoelastic parameters on both the stationary and overstable convection in porous media have been investigated for a horizontal porous layer [3] and for a vertical porous cylinder [4]. In the present paper, we investigate the convective instability of a viscoelastic fluid between two porous coaxial rotating cylinders. We determine how rotation and viscoelasticity of the fluid effect the critical Rayleigh number and corresponding mode maps.

FORMULATION OF THE PROBLEM

Consider an annular cylindrical cavity with height h^*, inner radius r_i^* and outer radius r_o^*. The cavity contains a porous medium with permeability K and porosity ϕ, and is saturated with a viscoelastic fluid with relaxation time λ^* and retardation time ε^*. The vertical sidewalls are insulated, the top and bottom walls are kept at constant temperatures T_1 and T_0, and all the walls are impermeable to flow. Meanwhile, the porous coaxial cylinders rotate about a vertical axis with a uniform angular speed $\mathbf{\Omega}$. If we limit the outer radius r_o^* to a length satisfying $r_o^* \ll g/\Omega^2$, then one can assume the gravity buoyancy is dominant and the centrifugal buoyancy is negligible [1]. Extending the Darcy-Jeffrey model to only include the Coriolis force and employing the Boussinesq approximation [3], one can obtain the linearized dimensionless governing equations for the system as:

$$\big(1+\varepsilon\frac{\partial}{\partial t}\big)^2\nabla^2 w - Ra\big(1+\varepsilon\frac{\partial}{\partial t}\big)\big(1+\lambda\frac{\partial}{\partial t}\big)\nabla_1^2\theta + Ta\big(1+\lambda\frac{\partial}{\partial t}\big)^2\frac{\partial^2 w}{\partial z^2} = 0, \tag{1}$$

$$\frac{\partial \theta}{\partial t} - w = \nabla^2\theta \tag{2}$$

where we have used h^*, h^*/κ (κ the thermal diffusivity) and T_0-T_1 to scale the length, time and temperature. θ and w are the deviation of the temperature and vertical velocity from the purely conductive basic state. ∇^2 is the Laplacian operator and ∇_1^2=$\nabla^2 - \partial^2/\partial z^2$. The Rayleigh Ra and Taylor number Ta are defined respectively as: $Ra = \frac{g\alpha K h^*(T_0-T_1)}{\nu\kappa}$, $Ta = \big(\frac{2\Omega K h^*}{\nu\phi}\big)^2$. A separable solution according to the normal mode analysis is given as:

$$(\theta, w) = [\Theta(z), \Psi(z)]\big[J_m(\gamma_{m,n}r)Y_m^{'}(\gamma_{m,n}r_i) - J_m^{'}(\gamma_{m,n}r_i)Y_m(\gamma_{m,n}r)\big]e^{\sigma t}\cos m\varphi \tag{3}$$

where $J_m(\cdot)$ and $Y_m(\cdot)$ are m-order Bessel functions of the first and second kind, respectively. $\gamma_{m,n}$ is the nth root of the following relation: $J_m(\gamma_{m,n}r_o)Y_m^{'}(\gamma_{m,n}r_i) - J_m^{'}(\gamma_{m,n}r_i)Y_m(\gamma_{m,n}r_o) = 0$. Taking the boundary conditions into account, one can get $\Theta(z)$=$C\sin\pi z$, where C is a constant. Then, we can obtain from Eqs. (1)-(3) that:

$$Ra = \frac{\pi^2+\gamma_{m,n}^2+\sigma}{\gamma_{m,n}^2}\Big[\frac{(\pi^2+\gamma_{m,n}^2)(1+\varepsilon\sigma)}{1+\lambda\sigma} + \frac{\pi^2 Ta(1+\lambda\sigma)}{1+\varepsilon\sigma}\Big]. \tag{4}$$

RESULTS AND DISCUSSION

In Eq. (4) σ is usually a complex number. When σ=0, the stationary convection occurs; when σ is a pure imaginary number, i.e. σ=$i\omega$ (ω>0), the oscillatory convection may occur [3]. For a pair of given radii r_i and r_o, the critical Rayleigh

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number R_c is found by evaluating Eq. (4) for each convective mode (m,n) in the integer space, i.e. $R_c = \min_{m,n} Ra$. We use the symbols R_c^O and R_c^S to denote the critical Rayleigh numbers for stationary and oscillatory convection, respectively. A pair of given radii corresponds to a unique critical Rayleigh number and a unique pair of (m,n), and in such case the convection is called a (m,n) preferred mode convection. We note that two necessary conditions for oscillatory convetion to occur must be satisfied: (i) ω>0; (ii) R_c^O<R_c^S.

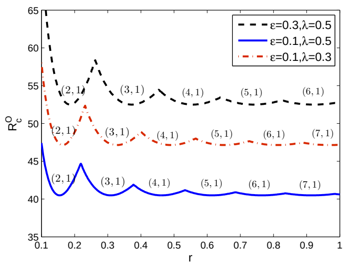

Figure 1. The effects of ε and λ on the R_c^O with r_o/r_i=3 and Ta=1.

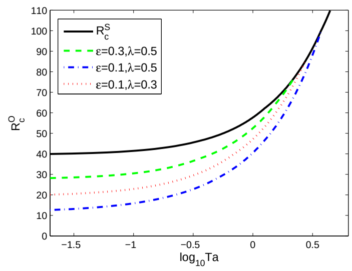

Figure 2. The effect of Ta on the R_c with r_i=0.5, r_o=1.5.

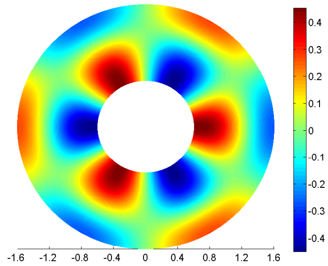

Figure 3. Temperature disturbance with mode $(3,2)$ for ε=0.3,λ=0.5 and Ta=2.

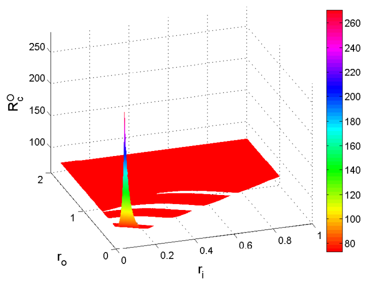

Figure 4. Marginal curve for the R_c^O with ε=0.3, λ=0.5 and $Ta = 2$.

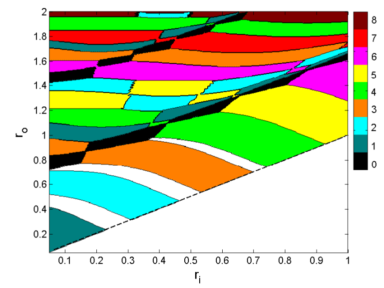

Figure 5. Mode map for the values of m with ε=0.3, λ=0.5 and $Ta = 2$.

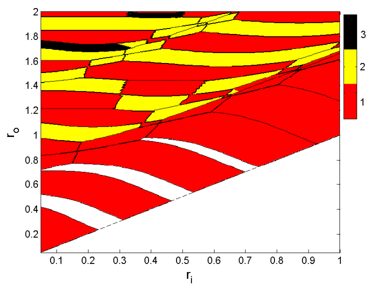

Figure 6. Mode map for the values of n with ε=0.3, λ=0.5 and $Ta = 2$.

Figure 1 shows the effect of viscoelastic parameters on the critical Rayleigh number for oscillatory convection. It can been seen that the increase in relaxation time λ reduces the R_c^O, which means that the elasticity of a viscoelastic fluid promotes the oscillatory convection. In contrast, the retardation time ε can suppress the oscillatory convection. Figure 2 gives the effect of rotation on the R_c. Obvioully, the R_c^O and R_c^S increase with the increase in Ta, so the rotation has a stabilizing effect on the onset of convection. Furthermore, a certian critical Taylor number exists for each curve of the oscillatory convection so that when Ta is greater than this critical value the oscillatory motion disappears.
The classical pattern of the disturbed temperature for oscillatory convection at the half height of the cylinder is presented in Figure 3. The preferred convective mode is $(3,2)$ mode. It can been seen that there are six adjacent convection cells in the azimuthal direction and one convection cell in the radial direction. We plot the mode maps for the values of m and n in a area of $[0,1]\times[0,2]$ in Figures 5 and 6. Figure 4 is the marginal curve for the R_c^O corresponding to mode maps in Figures 5 and 6. The appearance of white narrow bands in Figure 4-6 agrees with the effect of Ta shown in Figure 2, which results from the fact that the condition R_c^O<R_c^S in those regions is not satisfied.

CONCLUSIONS

The onset of oscillatory convection in a bounded annular rotating cylindrical porous cavity containing a viscoelastic fluid has been analyzed. The effects of rotation and viscoelastic parameters on critical Rayleigh number are discussed.

Acknowlegememt

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