AN INTEGRAL EQATION APPROACH TO MAGNETIC RECONNECTION

Mingtian Xu[1]
Institute of Thermal Science and Technology, Shandong University, Jinan,250061, China

Summary In the present work, an integral equation approach is developed to simulate the incompressible two-dimensional magnetohydrodynamic systems in general and magnetic reconnections in particular. Numerical experiments with the Orszag-Tang vortex and the doubly periodic coalescence instability show a good agreement with the available results obtained by other methods.

INTRODUCTION

Magnetic reconnection involves the breaking and mending of magnetic field lines. It may occur in solar flares, the magnetosphere of the earth, dynamo action, accretion disks, and tokamak plasmas. Therefore it is of great importance to investigate the magnetic reconnection phenomena. In Refs.[1-3], it was shown that methods based on integral equations are a promising alternative to conventional numerical methods for simulating dynamo actions, which inspires us to develop an integral equation approach to investigating the magnetic reconnection processes.

INTEGRAL EQUATION METHOD

GOVERNING EQATIONS

Many important physical processes may be described by the two-dimensional incompressible "reduced" magnetohydrodynamic equations. These equations can be formulated mathematically as follows:

$$\frac{\partial \nabla^2\phi}{\partial t}+[\nabla^2\phi,\phi]=[\nabla^2\psi,\psi]+\nu\nabla^4\phi\,,\ \frac{\partial\psi}{\partial t}+[\psi,\phi]=\eta\nabla^2\psi \tag{1}$$

with $\vec{B}=\vec{e}_z\times\nabla\psi, \vec{v}=\vec{e}_z\times\nabla\phi,\ j=\nabla^2\psi,\omega=\nabla^2\phi$, here $\vec{B}$ and $\vec{v}$ are respectively the magnetic field and the velocity field, ν and η are respectively the kinematic viscosity and magnetic resistivity, ϕ and ψ are respectively the streamfunction and magnetic flux function, $\vec{e}_z$ is the unit vector in the z direction, ∇ is the gradient operator, j is the current in the z direction, ∇^2 is the Laplace operator, [f, g] is the Poisson-bracket defined by $[f,g]=\partial f/\partial x\partial g/\partial y-\partial f/\partial y\partial g/\partial x$. The doubly periodic domain under consideration is $[0, L_x]\times[0, L_y]$. The boundary conditions is given by

$$\phi(x+L_x,y)=\phi(x,y),\phi(x,y+L_y)=\phi(x,y)\,,\quad \psi(x+L_x,y)=\psi(x,y),\psi(x,y+L_y)=\psi(x,y) \tag{2}$$

The initial conditions are prescribed as: $\phi(x,y,0)=\phi_0(x,y),\psi(x,y,0)=\psi_0(x,y)$

NUMERICAL METHOD

In order to get the Green functions G_ϕ and G_ψ we first attempt to analytically solve the following equations,

$$\frac{\partial\nabla^2 G_\phi}{\partial t}=\nu\nabla^4 G_\phi+\delta(x-x_0)\delta(y-y_0)\delta(t-t_0)\,,\ \frac{\partial G_\psi}{\partial t}=\eta\nabla^2 G_\psi+\delta(x-x_0)\delta(y-y_0)\delta(t-t_0) \tag{3}$$

Concerning the boundary conditions (2), we may expand the two Green functions and the Delta functions into Fourier series and submit them into equations (3) . By using the orthogonality of the Fourier series, we obtain

$$G_\phi=\sum_{m,n}c_{1mn}\exp[-c_{mn}(t-t_0)]\exp[-i(2m\pi x_0/L_x+2n\pi y_0/L_y)+i(2m\pi x/L_x+2n\pi y/L_y)] \tag{4}$$

$$G_\psi=\sum_{m,n}c_3\exp[-c_{2mn}(t-t_0)]\exp[-i(2m\pi x_0/L_x+2n\pi y_0/L_y)+i(2m\pi x/L_x+2n\pi y/L_y)] \tag{5}$$

where $c_{1mn}=-1/[L_xL_y(4m^2\pi^2/L_x^2+4n^2\pi^2/L_y^2)], c_{mn}=\nu(4m^2\pi^2/L_x^2+4n^2\pi^2/L_y^2)$, $c_3=1/L_xL_y$, $c_{2mn}=\eta(4m^2\pi^2/L_x^2+4n^2\pi^2/L_y^2)$. By the Green functions G_ϕ and G_ψ, the partial differential equations (1) can be changed to the following integral equations:

$$\phi(x,y)=\int_0^t\int_0^{L_x}\int_0^{L_y}G_\phi(x,y;x_0,y_0)\{[\nabla^2\psi,\psi]-[\nabla^2\phi,\phi]+\phi_0\delta(t_0)\}dy_0dx_0dt_0 \tag{6}$$

[1] Corresponding author. Email:mingtian@sdu.edu.cn

$$\psi(x,y)=\int_0^t\int_0^{L_x}\int_0^{L_y}G_\psi(x,y;x_0,y_0)\{-[\psi,\phi]+\psi_0\delta(t_0)\}dy_0dx_0dt_0 \tag{7}$$

Submitting Eqs.(4) and (5) into Eqs. (6) and (7) yields the following integral equations with respect to the time *t*,

$$\phi_{mn}(t)=\int_0^t\exp(-c_{mn}(t-t_0))\,f_{mn}^{\phi}dt_0+c_{1mn}\phi_{0mn}\exp(-c_{mn}t) \tag{8}$$

$$\psi_{mn}(t)=\int_0^t\exp(-c_{2mn}(t-t_0))\,f_{mn}^{\psi}dt_0+c_3\psi_{0mn}\exp(-c_{2mn}t),(m,n=1,2,3,\cdots\cdots) \tag{9}$$

where f_{mn}^{ϕ} and f_{mn}^{ψ} are functions of ϕ_{ij} and ψ_{ij} (*i,j=1,2,3,…*), $\phi_{0mn}=\int_0^{L_x}\int_0^{L_y}\exp(-i(2m\pi x/L_x+2n\pi y/L_y))\phi_0dydx$, $\psi_{0mn}=\int_0^{L_x}\int_0^{L_y}\exp(-i(2m\pi x/L_x+2n\pi y/L_y))\psi_0dydx$.

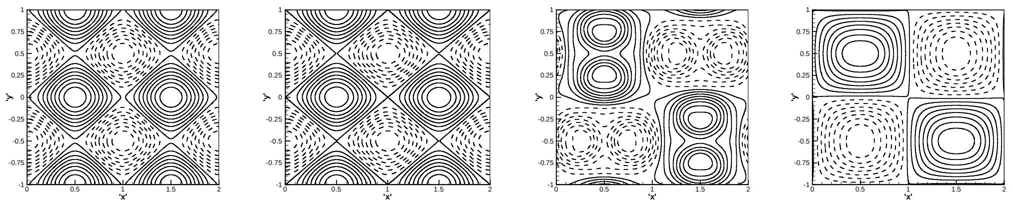
The TVD Runge-Kutta method is employed to solve the integral equation systems (8) and (9).

NUMERICAL RESULTS

We first apply the integral equation method to investigate the coalescence instability developing from the initial conditions,

$$\psi_0=\sin(\pi(x+y))\sin(\pi(x-y)),\phi_0=10^{-4}\exp(-10(x^2+y^2))$$

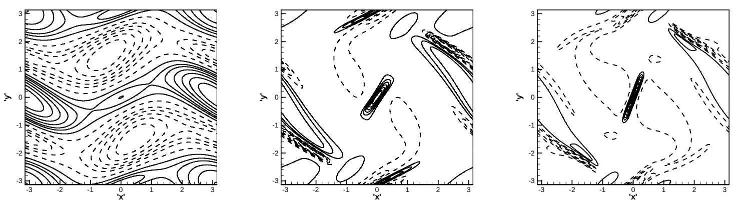
in the doubly periodic domain $-1\le x,y\le 1$. The contours of the magnetic flux function at various times are illustrated in Fig.1 for $\nu=0.004$ and $\eta=0.001$, which shows a good agreement with the results in [4].



(a) t=1.20 (b) t=1.75 (c) t=2.40 (d) t=3.75

Fig. 1 Variation of the magnetic flux function ψ with respect to the time *t* for the doubly periodic coalescence instability with $\nu=0.004$ and $\eta=0.001$. Positive contours are solid lines; negative contours are dotted lines.

Next we consider the Orszag-Tang vortex which is a two-dimensional flow evolving from the initial conditions, $\phi=2(\cos(x)-\cos(y)),\psi=2\cos(x)-\cos(2y)$. The contours of the current at various times for the Orszag-Tang vortex are depicted in Fig.2, which are in a good agreement with the results in [4].



(a) t=0.2 (b) t=0.6 (c) t=0.85

Fig. 2 Current for the Orszag-Tang vortex with $\nu=\eta=0.02$

CONCLUSTIONS

The integral equation method developed in this paper is capable for simulating the magnetic reconnections and other magnetohydrodynamic processes. Note that this method can be extended to deal with the three-dimensional magnetic reconnection and other magnetohydrodynamic phenomena.

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