CENTRIFUGAL THERMOMAGNETIC CONVECTION IN AN ANISOTROPIC POROUS MEDIUM

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Summary This work concerns with the onset of thermomagnetic convection induced by centrifugal acceleration in a magnetic fluid filled porous medium. The layer is assumed to exhibit mechanical as well as thermal anisotropies. It is found that the magnetic field has a destabilizing effect and can be suitably adjusted depending on the anisotropy parameters to enhance convection. The effect of anisotropies of the magnetic fluid filled porous medium is qualitatively different from that of ordinary fluid filled porous medium.

INTRODUCTION

Convection induced by centrifugal acceleration arises as an important mechanism dominating the gravitational one in several rotating fluid systems. Flows of this type and their stability characteristics have been reviewed by Vadasz [1]. Magnetic fluids differ from ordinary fluids by showing magnetic and as well as flow properties. Convection can also originate in these fluids due to the temperature dependence of their magnetization. Recently we have investigated thermomagnetic instability in magnetic fluid saturated porous media using linear and energy analyses [2,3].
One of the surprising results is that the overall thermal conductivity of a porous medium strongly depends on the internal heat transfer across the pore surfaces and thus behaves as a structural property rather than an intrinsic thermal property of the material [4]. Moreover polydispersity of magnetic fluids appears naturally since the particles in all commercially manufactured magnetic fluids form chain like aggregates called 'micelles'. In the presence of strong external body force gradients these aggregates make the volume fraction within the magnetic fluid a nonuniform one and in turn alter the overall thermal conductivity [5]. Hence this paper determines stability characteristics for the setting up of centrifugal convection in a magnetic fluid filled porous medium which is anisotropic permeability and thermal conductivity.

PROBLEM FORMULATION

A differentially heated magnetic fluid saturated anisotropic porous layer $-L/2<x<L/2$ is considered (see Fig.1 of [2].) It is subjected to a constant rotation rate ω about a vertical axis. A constant uniform external magnetic field $\overline{H}_o$ is applied parallel to the *x*-direction. Free convection occurs as a result of the centrifugal body force while the gravitational force is neglected. The conservation equations under the Boussinesq approximation are

$$\nabla \cdot \bar{v} = 0 \tag{1}$$

$$\frac{\mu}{\tilde{K}_p}\bar{v} = -\nabla p + \mu_0 M \nabla H - \rho\omega^2 x\hat{i} \tag{2}$$

$$\varepsilon\left(\rho C_{V,H} - \mu_0 H\frac{\partial M}{\partial T}\right)\frac{dT}{dt} + (1-\varepsilon)(\rho C)_S\frac{\partial T}{\partial t} + \mu_0 T\frac{\partial M}{\partial T}\frac{dH}{dt} = \nabla\cdot\left(\tilde{\lambda}\cdot\nabla T\right) \tag{3}$$

where $\tilde{K}_p = K_x\hat{i}\hat{i} + K_z\left(\hat{j}\hat{j} + \hat{k}\hat{k}\right)$ is the permeability, $\tilde{\lambda} = \lambda_x\hat{i}\hat{i} + \lambda_z\left(\hat{j}\hat{j} + \hat{k}\hat{k}\right)$ the thermal conductivity and ε the porosity. Since the applied magnetic field is static and the magnetic fluid is not electrically conducting, the electric field vector vanishes. The appropriate Maxwell's equations are

$$\nabla\cdot\overline{B} = 0\ ,\ \nabla\times\overline{H} = 0\ ,\ \overline{B} = \mu_0(\overline{H} + \overline{M}) \tag{4}$$

The relaxation time of the magnetization is assumed to be so small. The density and magnetic equations of state are

$$\rho = \rho_o[1 - \alpha(T - T_o)]\ ,\ M = M_o + \chi(H - H_o) - K(T - T_o) \tag{5}$$

STABILITY ANALYSIS

Following [2], we study the stability of the following quiescent basic state

$$\bar{v}_b = 0,\ T_b = T_o + \beta x,\ \overline{H}_b = \left(H_o + \frac{K_x\beta x}{1+\chi}\right)\hat{i}, \overline{M}_b = \left(M_o - \frac{K_x\beta x}{1+\chi}\right)\hat{i}. \tag{6}$$

using the method of small perturbations. The amplitudes of perturbations are governed by

$$(D^2 - K_r k^2)U - \left[Rm + Rc\left(x + \frac{1}{2}\right)\right]K_r k^2\Theta + RmK_r k^2 D\Phi = 0 \tag{7}$$

$$[(D^2 - \lambda_r k^2) - \Pr\sigma]\Theta - U = 0 \tag{8}$$

$$(D^2 - M_1 k^2)\Phi - D\Theta = 0 \tag{9}$$

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where U, Θ, Φ are the respective amplitudes of x-component of perturbed filtration velocity, temperature, magnetization, $k=(k_y^2+k_z^2)^{1/2}$ the wave number and $D\equiv d/dx$. The relevant nondimensional parameters are $Rm=\rho C_1\mu_0 K^2\beta^2 K_x L^2/\lambda_x\mu N(1+\chi)$ the magnetic Rayleigh number, $Rc=\rho C_1\alpha\beta\omega^2 K_x L^3/\lambda_x\nu N$ the centrifugal Rayleigh number, $Pr=\rho C_1\nu/\lambda_x$ the Prandtl number, $M_1=(1+M_0/H_0)/(1+\chi)$ the nonlinearity in magnetization, $K_r=K_z/K_x$ the anisotropy parameter of permeability and $\lambda_r=\lambda_z/\lambda_x$ the anisotropy parameter of conductivity. The vanishing velocity, temperature and magnetic potential perturbations at the sidewalls serve as the boundary conditions. The governing system can be expressed into a self-adjoint form and hence the onset of the stationary mode ($\sigma=0$) alone is discussed. A 12th order Galerkin method is used to solve the eigenvalue problem.

RESULTS AND DISCUSSION

In the case of ordinary fluids, i.e., when Rm=0 and $M_1\rightarrow\infty$ (7)-(9) reduce to a single equation with λ_r/K_r as the only parameter representing the anisotropy of the porous medium. The marginal curves are displayed in Fig.1 for different values of λ_r and K_r keeping λ_r/K_r=1. We see that a combined increase in the anisotropy parameters enhances the onset of convection and increases the cell size at the secondary state. This is clear from the streamlines and isotherms which are exhibited at the threshold. Figure 2 the effect of Rm on the onset condition. In all cases an increase in Rm produces an additional body force (Kelvin force) acting towards the hot boundary. This in turn combines with the existing centrifugal force and augments heat transfer across the layer via convection mechanism. Thus Rc_c decreases and becomes zero at a finite Rm depending on λ_r and K_r. The magnetic body force alone can induce instability above this Rm. We also observe that that λ_r/K_r increases Rc_c and hence suppresses the instability. Moreover it is noticed that Rc_c is determined by λ_r/K_r alone in the lower Rm region whereas it depends independently on the values λ_r and K_r in the higher Rm region. We see that larger values of λ_r and K_r with fixed λ_r/K_r decreases both Rc_c and k_c. In other words their effect is to destabilize the basic state accompanied by an increase in the cell size. This effect becomes significant for higher values of λ_r/K_r.

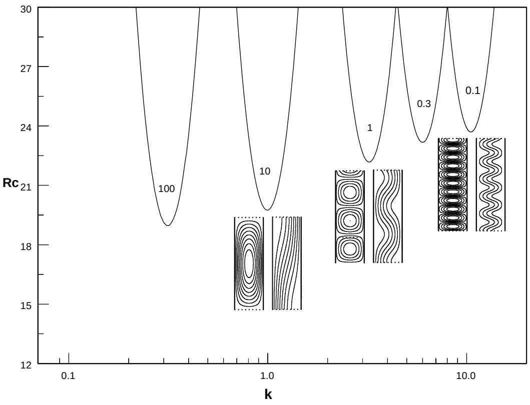

Fig.1

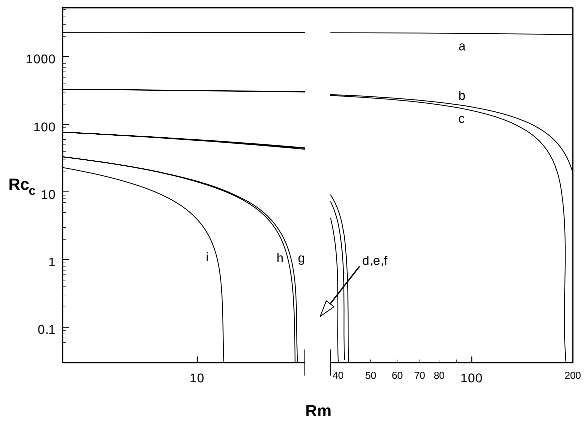

Fig.2

Fig.1 Marginal curves for different values of λ_r (=K_r) when Rm = 30, M_1 = 10 and λ_r/K_r = 1 with streamlines and isotherms at the threshold.

Fig.2 Rc$_c$ for various combinations of λ_r, K_r and λ_r/K_r when M_1=10: **a**: 10, 0.1, 100; **b**: 1, 0.1, 10; **c**: 10, 1, 10; **d**: 0.1, 0.1, 1; **e**: 1, 1, 1; **f**: 10, 10, 1; **g**: 0.1, 1, 0.1; **h**: 1, 10, 0.1; **i**: 0.1, 10, 0.01

CONCLUSIONS

The magnetic mechanism produces an additive effect on the setting up of secondary state. It starts dominating the centrifugal mechanism and becomes the only cause for instability if the magnetic field strength is increased beyond a particular level, which depends on both anisotropy ratios. Larger values of λ_r and K_r with fixed λ_r/K_r enhance the onset condition and increase the cell size at the threshold.

Acknowledgement: This work was supported by University Grants Commission, India through DRS SAP.

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