DEFLECTION OF LAMINAR LIQUID METAL FLOW BY A MAGNETIC POINT DIPOLE

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Summary We study distortion of laminar liquid metal flow by a magnetic point dipole in a straight square duct. This basic configuration is of fundamental interest for Lorentz force velocimetry, where the Lorentz force opposing the relative motion of conducting medium and magnetic field is measured to determine the flow velocity. The total force is highly dependent on the velocity profile, which changes its shape due to the acting Lorentz force itself. We are interested in the deflection of the flow and its dependence on magnitude and distribution of the magnetic field. To this end, we perform direct numerical simulations with an accurate finite-difference scheme in the limit of small magnetic Reynolds numbers. The hydrodynamic Reynolds number is choosen to be high enough to allow the generation of vortices and turbulent structures.

One challenge in metallurgical application is the velocity measurement of liquid metals, which are hot and agressive. A solution may be found in Lorentz force velocimetry (LFV), a contactless method for velocity measurements in conducting bodies [1, 2, 3]. The magnetic field of a permanent magnet induces currents in the conducting liquid. These introduce a braking force on the metal, that depends on the velocity profile and the magnitude and shape of the magnetic field. By measuring the opposite drag force on the magnet the mean velocity can be calculated.

Due to the braking force, the fluid is deflected and the velocity profile changes. Thus the deflection and the corresponding changes in the Lorentz force should be investigated. Previous works showed the magnetic obstacle effect, in particular the apperance of a vortex street behind an extended permanent magnet [4], and considered a quasi-2D approximation for a magnetic point dipole [5]. Our aim is to investigate the influence of a magnetic point dipole on liquid metal duct flow.

We consider a straight duct with square cross section $2L \times 2L$. The magnetic point dipole is located at a distance h to the surface and produces a magnetic induction $\mathbf{B}$, which is in nondimensional unit given by the dipole formula

$$\mathbf{B} = \alpha \left(\frac{3\mathbf{m}\cdot\mathbf{r}}{r^5}\mathbf{r} - \frac{\mathbf{m}}{r^3} \right), \tag{1}$$

with the normalisation coefficient α, $\mathbf{r}$ being the distance to dipole and $\mathbf{m}$ denotes the magnetic moment.

Two nondimensional parameters determine the bevavior of the flow: the hydrodynamic Reynolds number $\mathrm{Re} = \frac{\bar{u}\cdot L}{\nu}$, with the mean velocity $\bar{u}$ and the viscosity ν, and the Hartmann number $\mathrm{Ha} = B_{max} L \sqrt{\frac{\sigma}{\rho\nu}}$, where B_{max} is the maximum of the magnitude of the magnetic field inside the fluid, σ is the conductivity and ρ is the density of the liquid metal. The magnetic Reynolds number $\mu_0 \sigma \bar{u} L$ is assumed to be small.

The influence of the magnetic field on a laminar inflow is studied by direct numerical simulations with a highly conservative finite-difference method on a structured mesh [6]. We solve the incompressible Navier-Stokes equations

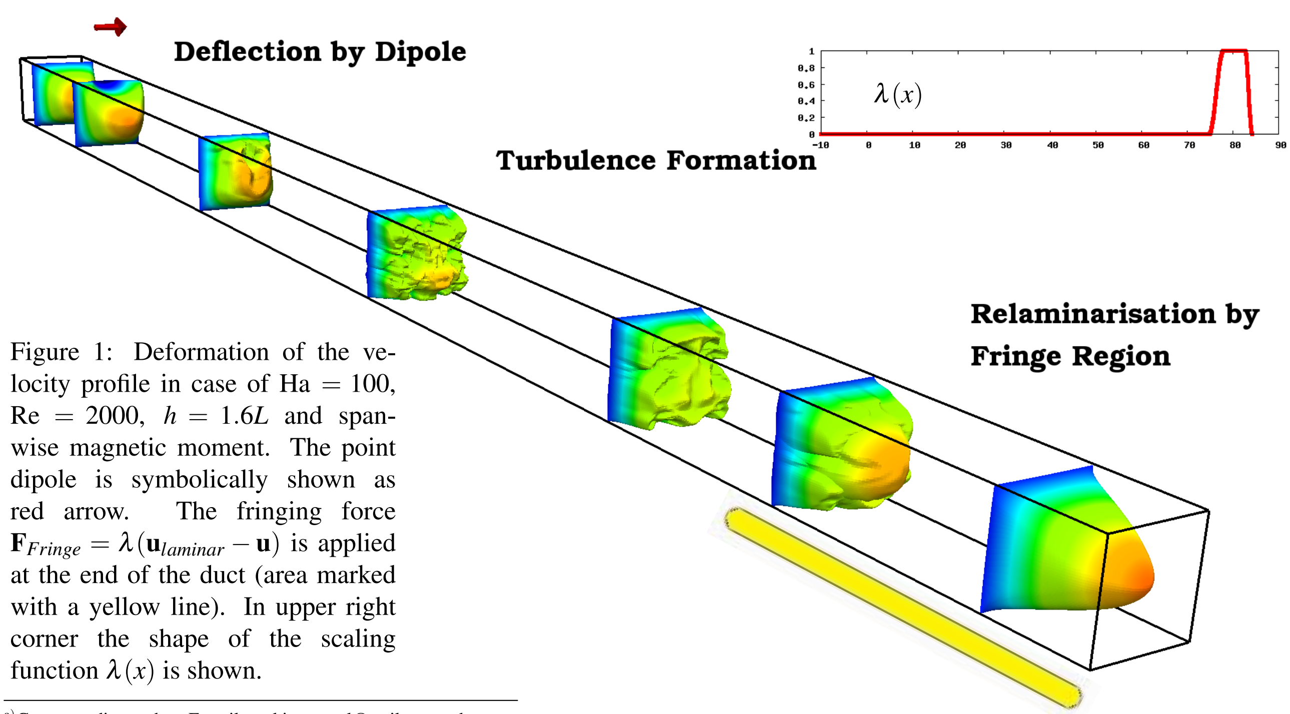

Figure 1: Deformation of the velocity profile in case of $\mathrm{Ha} = 100$, $\mathrm{Re} = 2000$, $h = 1.6L$ and spanwise magnetic moment. The point dipole is symbolically shown as red arrow. The fringing force $\mathbf{F}_{Fringe} = \lambda(\mathbf{u}_{laminar} - \mathbf{u})$ is applied at the end of the duct (area marked with a yellow line). In upper right corner the shape of the scaling function $\lambda(x)$ is shown.

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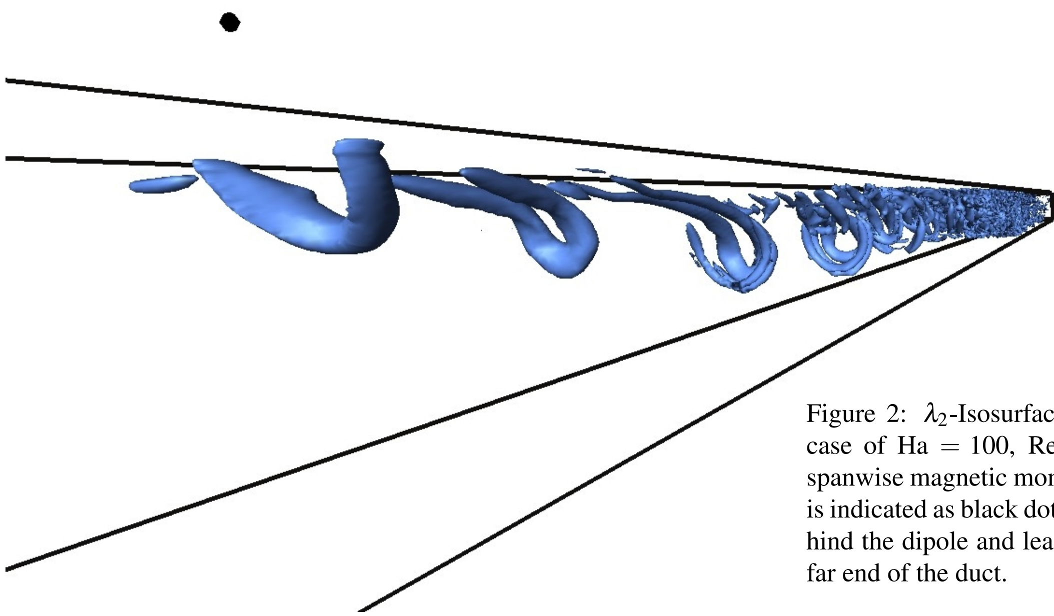

Figure 2: λ_2-Isosurfaces for $\lambda_2 = -1.0$ in the case of $\mathrm{Ha} = 100$, $\mathrm{Re} = 2000$, $h = 1.6L$ and spanwise magnetic moment. The dipole position is indicated as black dot. The hairpins appear behind the dipole and lead to turbulent flow at the far end of the duct.

$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{u} + \frac{\mathrm{Ha}^2}{\mathrm{Re}}(\mathbf{j}\times\mathbf{B}), \quad \nabla\cdot\mathbf{u} = 0, \tag{2}$$

where $\mathbf{u}$ is the velocity and p is pressure. The induced currents $\mathbf{j}$ are calculated with the help of Ohm's law, $\mathbf{j} = -\nabla\varphi + \mathbf{u}\times\mathbf{B}$, and the Poisson equation for the electric potential φ by $\Delta\varphi = \nabla\cdot(\mathbf{u}\times\mathbf{B})$. The walls are insulating and satisfy the no-slip boundary condition. In streamwise direction, periodic boundary conditions are used. This allows us the application of Fast Fourier Transform and, thereby, increases the speed of the code. To relaminarize the velocity profile we take advantage of an artificial volume force $\mathbf{F}_{Fringe} = \lambda(\mathbf{u}_{laminar} - \mathbf{u})$ with $\lambda(x)$ as shown in figure 1 [7].

Several simulations are done on $4096\times128\times128$ grid points for a duct of $30\pi L\times 2L\times 2L$. The hydrodynamic Reynolds number is set to 1000, 2000 and 3000. The dipole orientation is choosen to be either vertical, streamwise or spanwise. For $\mathrm{Re} = 1000$ all settings remained laminar, while for higher Reynolds number vortex shedding is observed for one dipole orientation. An significant deflection of the flow is found for Hartmann number $\mathrm{Ha} = 100$ and dipole distance $h = 1.6L$.

For vertical and streamwise oriented dipole, the flow remains laminar. Below the dipole position two vortices are formed. Additionally Hartmann layers are observed in certain areas of the top surface, which seem to damp the vortices. In case of the spanwise oriented dipole the area of Hartmann layers are smaller and close to the side walls. This results in vortex shedding and the flow becomes turbulent (figure 1 and 2). The λ_2-method shows the creation of hairpin vortices behind the dipole. The transient behaviour of Lorentz force shows a periodic oscillation in tune with the shedding of the vortices.

Recent investigations include a finer parameter study, i. e. the influence of other orientations and the distance of the dipole. Finally a quantitive evaluation of momentum balance, wall stresses and flow deformation is in progress.

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References

[1] Thess A., Votyakov E., Kolesnikov Y.: Lorentz Force Velocimetry, *Phys. Rev. Lett.*, 2006.
[2] Thess A., Votyakov E., Knaepen B., Zikanov O.: Theory of the Lorentz force flowmeter, *New J. Phys.*, 2007
[3] Kirpo M., Tympel S., Boeck T., Krasnov D., Thess A.: Electromagnetic Drag on a Magnetic Dipole near a Translating Conducting Bar, *J. Appl. Phys.*, 2011.
[4] Votyakov E. V., Zienicke E. A., Kolesnikov Y.: Constrained flow around a magnetic obstacle, *J. Fluid Mech.*, 2008.
[5] Cuevas S., Smolentsev S., Abdou M.: Vorticity generation in creeping flow past a magnetic obstacle, *Phys. Rev. E*, 2006.
[6] Krasnov D., Zikanov O., Boeck T.: Comparative study of finite difference approaches in simulation of magnetohydrodynamic turbulence at low magnetic Reynolds number, *Computers and Fluids*, 2011.
[7] Nordström J., Nordin N., Henningson D.: The Fringe Region Technique and the Fourier Method Used in the Direct Numerical Simulation of Spatially Evolving Viscous Flows, *SIAM J. Sci. Comput.*, 1999.