# INSTABILITY OF UNSTEADY STREAKS WITH STREAMWISE MAGNETIC FIELD

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*Summary* The instability of unsteady streaky flow in a channel under the presence of a constant and uniform streamwise magnetic field is investigated with linear stability analysis. Without magnetic field, the linear secondary perturbations evolving on the basic state exhibit transient growth. The suppression of the transient growth of perturbations with different wavenumbers is investigated upon increasing the Hartmann number, and a simple power law is found between the optimal wavenumber and the Stuart number.

## INTRODUCTION

Instability and transition to turbulence in plane channel flow usually occurs in practice below the critical parameters (Reynolds numbers) predicted by classical linear stability analysis. In recent years, it has been recognized that the transient growth of linear perturbations may play a significant role in sub-critical transition [1]. Such perturbations are asymptotically stable, i.e. they decay at large times. Strong modification of the basic flow can be achieved due to significant amplification of initial primary perturbations, and then secondary perturbations may become rapidly amplified and thereby lead to transition. For plane channel flow, the strongest amplification is supplied by streamwise vortices, which interact with the mean flow and evolve into streamwise streaks. These streaks are observed in experiments and can be viewed as a key element in the transition scenario and the dynamic processes of self-sustaining turbulence. In the presence of streamwise magnetic field, the additional Lorentz force may affect the formation and breakdown of streaks and hence modify the transition. We study this topic by means of a transient linear perturbation analysis using a channel flow with unsteady streaks as the basic state.

## FORMULATION OF THE PROBLEM

The flow of an incompressible electrically conducting fluid in an infinite parallel plane channel between two insulating walls is considered. The flow is subjected to a constant and uniform streamwise magnetic field, and is driven by a variable pressure gradient such that constant mass flux is maintained. With the assumption of low magnetic Reynolds number, the nondimensional governing equations and boundary conditions are:

$$\frac{\partial \mathbf{v}}{\partial t}+\left(\mathbf{v}\cdot\nabla\right)\mathbf{v}=-\nabla p+\frac{1}{Re}\nabla^2\mathbf{v}+N\left(-\nabla\phi\times\mathbf{e}+\left(\mathbf{v}\times\mathbf{e}\right)\times\mathbf{e}\right) \tag{1}$$

$$\nabla\cdot\mathbf{v}=0 \tag{2}$$

$$\nabla^2\phi=\nabla\cdot\left(\mathbf{v}\times\mathbf{e}\right) \tag{3}$$

$$u=v=w=\frac{\partial\phi}{\partial z}=0\ at\ z=\pm 1 \tag{4}$$

Here $\mathbf{e}\equiv(1,\ 0,\ 0)$ and $x,\ y,\ z$ denotes streamwise, spanwise and wall normal direction respectively. The centerline velocity of the channel Poiseuille flow $U$, the half width of the channel $L$, and the imposed magnetic field strength $B$ have been taken for non-dimensionalization. Parameters in the equations are the Reynolds number $Re = UL/\upsilon$, and the magnetic interaction parameter, or the Stuart number $N = Ha^2/Re$, where $Ha = BL\ (\sigma/\rho\upsilon)^{1/2}$ is the Hartmann number.
The governing equations are linearized around unsteady streaks, $U(y,\ z,\ t)$, i.e. both the time dependence and the presence of spanwise and wall normal velocity components are taken into account. The linear perturbations are decoupled monochromatic Fourier modes in the homogeneous $x$-direction, thus the linear perturbations are written as $\mathbf{u}_p(x,\ y,\ z,\ t) = \mathbf{u}(y,\ z,\ t)\ exp(i\alpha x)$, where $\alpha$ is the streamwise wavenumber. An energy norm can be defined as $E(t) = (1/V)\int \mathbf{u}_p\cdot\mathbf{u}_p\,dV$, and the ratio between perturbation energy $E(T)$ at time $T$ and the initial perturbation energy $E(0)$ is called the amplification factor of perturbation energy, $G = E(T)/E(0)$.
Using a Lagrangian formalism, the maximum value of $G$ for given parameters $G_{max}(Re,\ Ha,\ T,\ \alpha)$ is determined via optimization constraints, which are enforced by the adjoint fields [1]. The optimal perturbation and amplification at time $T$ are obtained by an iterative scheme, in which forward integration of the linearized governing equation is followed by backward integration of the adjoint equations. The base streaky flow is computed using the direct numerical simulation (DNS) code from Ref. [2]. The initial primary optimal perturbations are calculated following the same procedure mentioned above, in which the Poiseuille flow is taken as basic state. They are streamwise vortices with $\alpha = 0$. The spanwise wavenumber $\beta$ is around 2.044, which is in a good agreement with Ref. [3]. A few subcritical Reynolds numbers are considered, namely $Re$ = 1000, 2000, 3000 and 5000. To quantify the primary disturbance field,

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an amplitude $A$ is defined as $A = E_0/E_B$, where $E_0$ is the initial perturbation energy at $t = 0$ and $E_B$ is the mean kinetic energy of the Poiseuille flow, and the relation of $ARe^2 \approx const.$ is maintained.

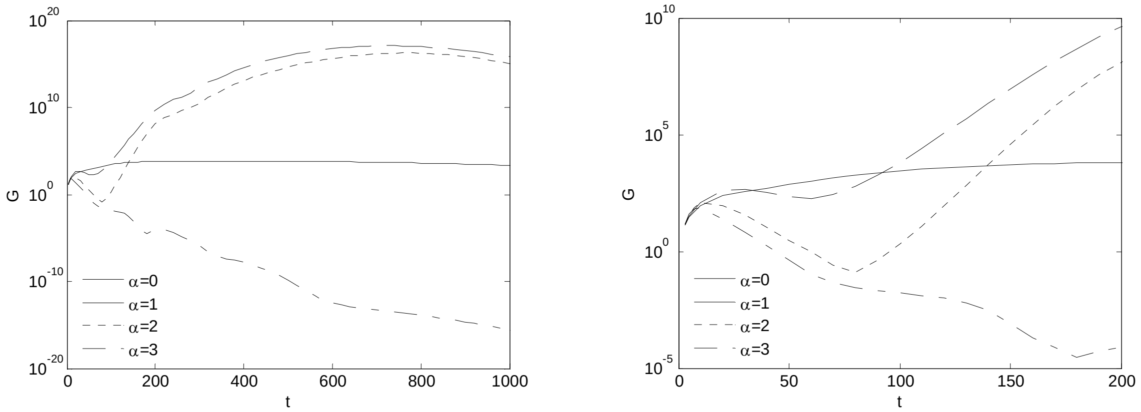


FIG. 1. Evolution of perturbations on top of unsteady steaks with amplitude $A = 10^{-4}$ at $Re = 3000$ (left). The short time behavior is shown in larger scale (right).

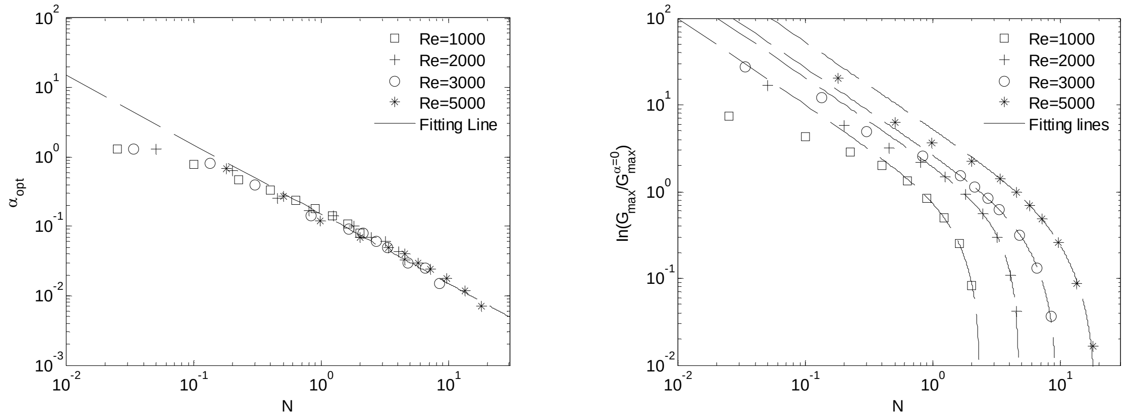


FIG. 2. Optimal streamwise wavenumber $\alpha_{opt}$ (left) and maximum value of amplification factor $G_{max}$ normalized with $G_{max}^{\alpha=0}$ (right) as function of the magnetic interaction parameter $N$.

## RESULTS

Compared with our previous study, in which steady streaks are investigated [4], the linear secondary perturbations evolving on the unsteady streaky flow exhibit only transient growth (Fig. 1). For larger wavenumbers ($\alpha = 3$ in Fig. 1), there is transient growth for short time followed by exponential decay. The appearance of a local maximum of amplification factor is identical to primary perturbations as the time dependent basic flow is still close to the Poiseuille flow at small times. The global maximum for larger times ($t > 600$ for $\alpha = 1$; $\alpha = 2$) can be attributed to the linear secondary perturbations evolving on fully developed, slowly decaying streaks. Both the optimal streamwise wavenumber $\alpha_{opt}$ and corresponding amplification factor are decreased when increasing $Ha$ until the least stable mode is streamwise independent, i.e., $\alpha = 0$. We interpret this as a complete suppression of secondary instability and associate it with a critical value $N_c$. For sufficiently large $N < N_c$, a simple scaling law between $\alpha_{opt}$ and $N$ is observed: $\alpha_{opt} \sim N^{-1}$. It can be interpreted as a balance between the Reynolds stress term and the Joule dissipation term in the perturbation kinetic energy budget. Using the same idea, one can arrive at a rough estimate $\text{In}(G_{max}/G_{max}^{\alpha=0}) \sim N^{-1}[a - b/(c + N^{-2})]$ with three parameters $a, b, c$. In Figure 2, good fits can be found in the range where $\alpha_{opt} \sim N^{-1}$ is valid. The approximation can be used to identify $N_c$ for suppression of secondary growth. It is found to increase with the Reynolds number. The same trend is observed in accompanying direct numerical simulations of relaminarization.

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