CODE DEVELOPMENT FOR NUMERICAL SIMULATION ON MAGNETOHYDRODYNAMIC RECTANGULAR DUCT FLOWS

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Summary Liquid metal flow in a vertical rectangular duct subjected to an strong external magnetic field is numerically simulated by three-dimensional SIMPLE algorithm on a finite volume structured staggered grid. The current density and the Lorentz force are calculated by solving the electrical potential equation at low magnetic Reynolds numbers and high Hartmann numbers with consistent and conservative scheme. The code is validated by calculating Hunt's case II. The numerical and analytical solution has a good agreement.

INTRODUCTION

The study of Magnetohydrodynamic (MHD) duct flow is significant in the application of fusion reactor liquid metal blankets such as self-cooled liquid metal blankets and dual-cooled liquid metal blanket [1] . MHD duct flow has been studied experimentally and theoretically extensively. The theoretical investigation has been performed by analytical method, matched asymptotic method and numerical method. Hunt [2] presented an analytical solution for fully developed rectangular duct flows with special wall boundary condition. Shercliff's and Hunt's case II have been widely used as benchmark problems to test numerical simulation code.

Three-dimensional numerical study for the MHD duct flow is difficult especially for High Hartmann numbers. As the thickness of Hartmann layer perpendicular to the applied magnetic field is the scale of Ha^{-1}, the layer is very thin in the application of fusion blankets. In the numerical simulation, there must be at least several grids in the Hartmann layer. Therefore it is hard to get a consistent and conservative numerical result for the electric current on an extremely non-uniform mesh system. Three-dimensional numerical calculation was conducted at a low Hartmann number or ignoring the viscous effect in the early stage of MHD three-dimensional numerical study. Ni[3, 4] designed a consistent, conservative and accurate scheme to calculate the current density and the Lorentz force by solving the electrical potential equation for MHD flow at low magnetic Reynolds numbers and high Hartmann numbers on a finite-volume structured collocated mesh, an arbitrary collocated mesh and a staggered mesh. The serial methods could solve the MHD duct flow with high Hartmann number and get a good accuracy. The methods have been widely used in the numerical study of MHD flow.

SIMPLE [5] type methods have second-order temporal accuracy for unsteady flows. The classical second-order projection method and SIMPLE type methods could be united with the general second-order projection formula.[6]

Formulation and boundary condition

The dimensionless governing equations for MHD flow with low magnetic Reynolds number are:

$$\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}=-\nabla p+Re^{-1}\nabla^2\mathbf{u}+N(\mathbf{J}\times\mathbf{B}),\quad \nabla\cdot\mathbf{u}=0,\quad \mathbf{J}=\sigma(-\nabla\phi+\mathbf{u}\times\mathbf{B}),\nabla\cdot\mathbf{J}=0,\quad \nabla\cdot(\sigma\nabla\phi)=\nabla\cdot\sigma(\mathbf{u}\times\mathbf{B}).$$

Here $\mathbf{u},\mathbf{B},\mathbf{J},p,\phi$ are velocity, external magnetic field, induced electric current, pressure, induced electric potential respectively. They are scaled with $v_0^*,B_0^*,\sigma_0^*v_0^*B_0^*,\rho^*v_0^{*2},a^*v_0^*B_0^*$, respectively. v_0^* is the average velocity, B_0^* is the applied external magnetic field, ρ^* is the density of the fluid, σ_0^* is the conductivity of the fluid, a^* is the half length of the duct along the external magnetic field. $Re=v_0^*a^*/\upsilon^*$ is Reynolds number, $N=a^*\sigma^*B_0^{*2}/\rho^*v_0^*$ is interaction parameter. Hartmann number is $Ha=a^*B_0^*\sqrt{\sigma^*/\rho^*\upsilon^*}=\sqrt{Re\cdot N}$, the square of the Hartmann number is the ratio of the electromagnetic forces to the viscous forces.

No-slip boundary condition is used the velocity at the solid wall. At the inlet a uniform velocity $w=1$ is given. At the outlet the velocity and pressure is fully developed. The pressure is extrapolated at the inlet and outlet. There is no induced electric current normal to the outside walls.

Numerical method

Fully staggered grid is used to arrange the velocity vector, pressure, electric potential, electric current density and Lorentz force. The pressure and electric potential is located at the centre of the grid. The velocity components u,v,w and the current density components j_x,j_y,j_z are arranged at the face of the control volume as shown in fig 1.

The momentum and mass conservative equations are solved by SIMPLE algorithm. The electric potential and current density is calculated using by consistent and conservative method. The detail calculation process and interpolation technique could be seen in reference[4].The pressure, diffusion and convection terms are discretised by a second order central difference and finite volume scheme. The pressure correction equation is derived from the mass conservative equation.

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The momentum and pressure correction equation are solved in fluid block. The electric potential are solved both in fluid area and Hartmann walls.

The momentum discretised equations are solved by the alternating direction implicit (ADI) method. The pressure correction and electric potential discretised equations are solved by Gauss-Seidel iteration method. The convergence criteria are set as that the maximum residual values of the mass and electric current conservative equation tend to zero.

Code validation

In order to test the code we use the program to simulate the three-dimensional rectangular MHD duct flow with insulating parallel wall and electrically conducting Hartmann walls (Hunt's case II). The dimensionless parameters are $Ha = 1000$, $Re = 1000$. The geometry dimension of the duct is $2\times2\times5$, the thickness of the Hartmann wall is 0.08, the conductivity of the Hartmann wall is 0.5. Therefore the wall conductance ratio is $c = \sigma_w^* h_w^* / \sigma_0^* a_0^* = 0.04$.

An extremely non-uniform mesh is generated automatically with Hartmann number sensitivity. The domain contains $48\times64\times50$ grids including Hartmann walls. The numerical results of the velocity w, the induced electric current at the outlet, pressure p and the maximum residual of the mass conservation $\delta_{\max} = (\nabla\cdot\mathbf{u})_{\max}$ are presented in fig 2-6. The velocity distribution and the induced electric current have a good agreement with the analytical solution. The residual of the mass conservation equation could tend to zero quickly.

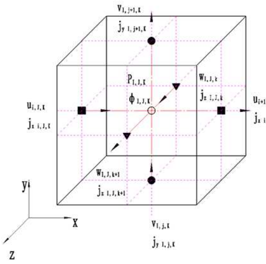

Fig 1 variables arrangement

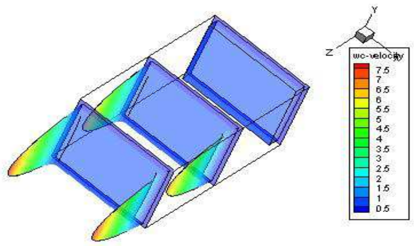

Fig 2 velocity distribution at the cross section

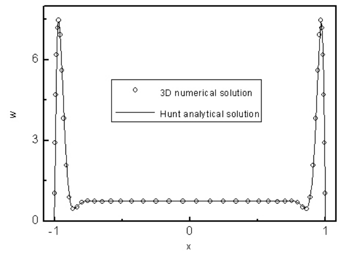

Fig3 velocity along x at outlet and $y = 0.089$

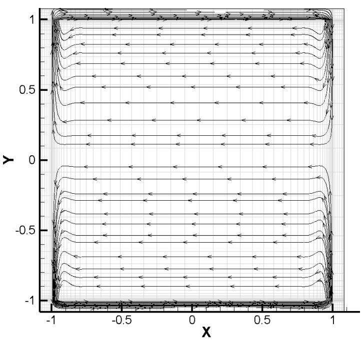

Fig4 induced electric current at the outlet

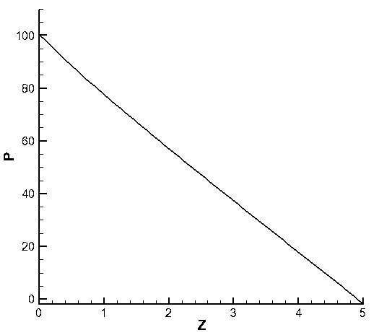

Fig 5 pressure along axial direction at x=0, y=0

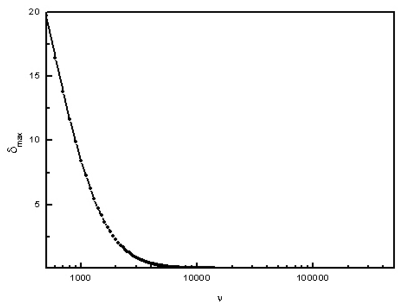

Fig 6 the maximum residual of the mass conservation

CONCLUSIONS

Three-dimensional numerical method to simulate the MHD rectangular duct flows with high Hartmann number and low magnetic Reynolds number on stagger grids are presented. The momentum and conservation equation are solved by SIMPLE type methods. The consistent and conservative scheme is applied to calculate the induced electric current and Lorentz force. The code is validated by comparing with the analytical solution of Hunt's case II. Numerical solution and analytical solution has a good agreement.

Acknowledgements

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