THE EFFECTS OF MEMBRANE LAWS ON THE CAPSULE FLOW IN MICROFLUIDIC CHANNEL WITH SQUARE CROSS-SECTION

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Summary The flow of a capsule suspension through a microfluidic pore with dimensions comparable with those of the suspended particles can be used to infer the elastic properties of the capsule membrane, provided a mechanical model of the process is available. We present a numerical model of the fluid-structure interactions in a square section pore and specifically consider the influence of a membrane law with strain softening or hardening properties. We use a novel numerical model that couples a boundary integral method for the internal and external fluid flows and a finite element method for the membrane deformation. This model is stable even when the membrane is under compression and tends to buckle. We find that the capsule deformation increases with the flow strength and/or the wall confinement. In the same conditions, the deformation is larger for strain softening than for strain hardening membranes.

INTRODUCTION

Suspensions of microcapsules consisting of a liquid droplets enclosed by impermeable membranes are common in nature (cells, eggs, vesicles) and in industry (cosmetic, pharmaceutical or food products). The membrane properties play an essential role in controlling the particle deformation or possible breakup, which is to be induced or prevented depending on the application. A simple way to measure the membrane properties is to flow a capsule suspension in a square cross section microfluidic pore with dimensions comparable with those of the suspended particles [1]. The hydrodynamic stresses and channel confinement cause *large* deformations of the capsules that depend on the flow strength, on the particle relative size compared to the channel side and on the membrane constitutive law. The capsule deformation, volume and velocity are measured simultaneously by means of image analysis. But in order to analyze the experimental data in terms of the capsule membrane properties, we need a model of the flow of a capsule in a square section pore. This situation has been considered recently [2], but has been limited to prestressed capsules, owing to numerical instabilities. We have designed a numerical model of the fluid-structure interactions leading to large capsule deformation in a square section pore. We specifically consider the influence of a membrane law with strain softening or hardening properties.

METHOD

Problem description

An initially spherical capsule (radius a) flows along the axis of a microfluidic channel with a square cross-section (side 2ℓ). The interior and exterior of the capsule are incompressible Newtonian fluids with the same density ρ and viscosity μ. Assuming low Reynolds number, the flow of the internal and external fluids are described by the Stokes equations. The thin wall of the capsule is an impermeable hyperelastic isotropic surface with surface shear modulus G_s and area dilation modulus K_s. Neglecting bending resistance, the membrane deformation occurs in-plane. We consider two membrane constitutive laws: the shear softening neo-Hookean law (NH) with $K_s/G_s = 3$ and the shear hardening Skalak law (SK) with $K_s/G_s = 1+2C$, where C is a parameter which increases with the resistance to area dilation. When $C = 1$, the two laws have the same small deformation behavior. The capsule motion and deformation are controlled by the membrane law and two main parameters: the size ratio a/ℓ between the capsule initial radius and the channel cross dimension and the capillary number $Ca = \mu V/G_s$ measuring the ratio between viscous and elastic forces (i.e. the relative flow strength) where V is the mean external flow velocity.

Numerical method

The fluid-structure coupling is implemented by means of a boundary integral method for the two fluid flows and a finite element method for the membrane [3]. The advantage is that we need to discretize only the boundaries of the flow domain, i.e. the capsule membrane and the channel wall, entrance and exit sections. This allows us to perform a Lagrangian tracking of the membrane points with high accuracy. The method has been shown to be stable even when the membrane is undergoing compression and tends to buckle.

COMPARISONS BETWEEN NEO-HOOKEAN AND SKALAK (C=1) MEMBRANE LAWS

Capsule profiles

The axial and cross profiles of a capsule are shown in Fig. 1. We note that the capsule undergoes large deformations from its initial spherical shape and takes a parachute shape with an inverted curvature at the back, except when the capillary number is very small. The overall shape is not axisymmetric as the cross profile tends to expand into the corners of the channel. As expected, the NH and SK profiles are almost superimposed for small deformation at low flow strength

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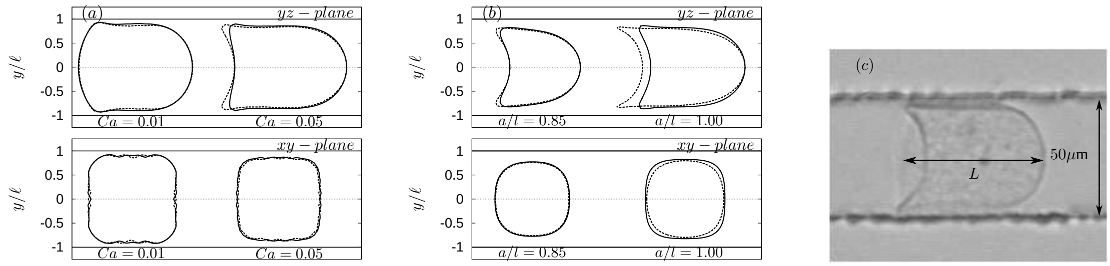

Figure 1. Comparisons of steady capsule profiles (solid line: SK law (C=1.0), dashed line: NH law). (a) Effect of Ca for $a/\ell = 1.1$. (b) Effect of a/ℓ for $Ca = 0.1$. (c) Experimental axial profile of a capsule ($a/\ell = 1.1$) with a cross-linked ovalbumin membrane [4].

($Ca = 0.01$). However, the NH capsule takes a parachute shape for $Ca = 0.05$, whereas the SK capsule still has a slug shape (Fig. 1a). As Ca increases, the SK capsule also takes a parachute shape (Fig. 1b). The flow strength level for which the curvature inversion appears thus depends on the membrane constitutive law. For high Ca, the deformation increases with the size ratio (Fig. 1b). We then note that either higher hydrodynamic forces (increase of Ca) or higher wall confinement constraints (increase of a/ℓ) cause a larger capsule deformation. For the same values of Ca and of a/ℓ, the deformation is larger for a strain softening NH capsule than for a strain hardening SK one. Parachute profiles are observed experimentally for capsules with a cross-linked ovalbumin membrane flowing in a microfluidic channel with a $50 \times 50\ \mu\text{m}^2$ cross-section (Fig. 1c).

Global effect of membrane law, flow strength and confinement

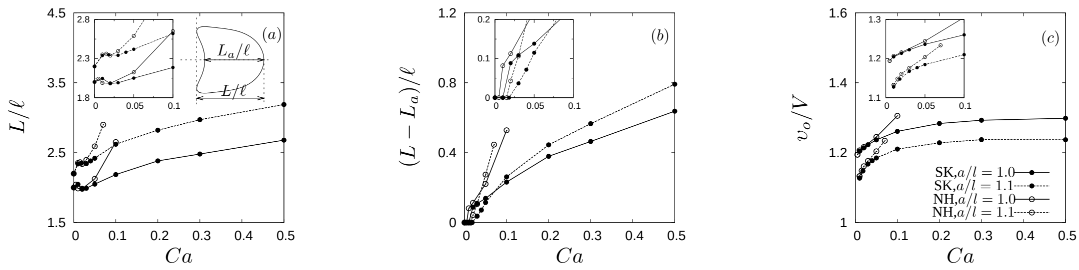

Figure 2. Effect of Ca, a/ℓ and membrane law on the capsule length L, parachute depth $L - L_a$ and center of mass velocity v_o.

The overall capsule deformation may be quantified by the maximum length L and the axial length L_a (Fig. 2a). Until a parachute is formed for $Ca = Ca_c$, there is little influence of the membrane law. We note that Ca_c is larger for a SK than a NH capsule. This is due to the strain softening property of the NH membrane which allows larger deformation for the same stress level than a strain hardening SK membrane. For $Ca > Ca_c$, the elongation of the NH capsule increases with Ca until a critical value above which no steady equilibrium state is reached (Fig. 2a,b), as was observed in a cylindrical pore [1]. In contrast, the elongation of a SK capsule increases with Ca without bound. This is due to the strain hardening property of the membrane which limits the deformation under an increasing external viscous load. The overall effect of the size ratio is to increase the deformation for a given flow strength. Finally, we note that the capsule velocity decreases when the deformation decreases, i.e. when the capsule tends to block the pore (Fig. 2c).

CONCLUSION

Since a capsule undergoes large deformation when it flows in a small pore, the effect of the membrane constitutive law is essential. In a square section microfluidic pore, the apparition of the parachute shape and the subsequent variation of $L - L_a$ with flow strength can be used to detect whether the membrane is strain hardening or softening. It is then possible to find the value of the membrane shear modulus for capsules flowing in a microfluidic pore, using a similar inverse analysis technique as the one designed for cylindrical pores [1].

References

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