OSCILLATORY ELECTROOSMOTIC FLOW THROUGH A CHANNEL WITH SLIPPING STRIPES ON WALLS

Henry C. W. Chu & Chiu-On Ng [a)]
Department of Mechanical Engineering, The University of Hong Kong, Pokfulam Road, Hong Kong

Summary This is a theoretical study on time-oscillating electroosmotic flow through a plane channel bounded by two parallel plates, which are micropatterned with periodic and inhomogeneous slipping and charged stripes aligned in a direction parallel or normal to the flow. Onsager relations for the fluid and current fluxes are deduced. Theoretical limits of two kinds of wall patterns are generalized to the oscillatory regime. Notable similarities and differences between the steady and oscillatory electroosmotic flow are revealed.

INTRODUCTION

Many studies have been carried out to look into the effect of boundary slip on electroosmotic flow (EOF) in microchannels. Not until recently has the focus been largely on surfaces with heterogeneous properties. In the limit of a very thin electric double layer (EDL), Squires [1] made two remarks for flows over inhomogeneously slipping surfaces. First, when the wall potential is everywhere constant, the EOF is enhanced by exactly the same amount as if the surface were uniformly slipping with slippage equal to the effective slip length. Second, surfaces with uncharged perfect-slipping regions will give rise to no enhancement due to the slip, instead giving precisely the same EOF as if the surface were completely non-slipping and homogeneously charged. Later studies extended the scope of the problem by considering arbitrary EDL thickness [2-5], partial slipping regions [4-5] and finite channel height [5].

These studies all pertain to steady EOF. In this paper, we examine the applicability of these remarks when the flow is oscillatory. We derive the Onsager constitutive equations for a channel with micropatterned surfaces where the periodic stripes are of distinct slippage and wall potential. The flow is driven by oscillatory mechanical and electrokinetic forcings of the same frequency. Flows that are longitudinal or transverse to the stripes are investigated. Our results show that the electrohydrodynamics under oscillatory forcing is not completely analogical to its steady counterpart.

PROBLEM FORMULATION

Consider longitudinal and transverse time-oscillating flows through a plane channel bounded by two identical superhydrophobic parallel plates, each with a periodic stripe pattern as shown in Fig. 1. The two walls are located at $y = \pm h$, where h is half the channel height and the patterns are aligned in-phase with each other. One unit of the wall pattern, which is of a period length $2L$, comprises a slipping stripe of width $2aL$, slip length λ and wall potential ζ_S, and a non-slipping stripe of width $2(1\text{-}a)L$ and wall potential ζ_{NS}. The number $0 \le a \le 1$ denotes the area fraction of the wall which is slipping. The intrinsic slip length is allowed to vary in the range $0 \le \lambda \le \infty$, where the two limits $\lambda = 0$ and $\lambda = \infty$ correspond to no slip and perfect slip conditions, respectively. The pattern period length is assumed to be comparable with the channel height. The fluid under consideration is a Newtonian, incompressible and symmetrical electrolyte of uniform dielectric constant ε and viscosity μ. We shall restrict our discussion to the low-Dukhin limit.

On assuming low potentials and non-overlapped EDLs, the Debye-Hückel approximation is invoked to linearize the Poisson-Boltzmann equation which governs the electric potential $\psi(x,y)$ due to the EDL. The total potential ψ_{tot} is the difference between the potential due to the EDL and that by the presence of an externally applied electric field $\bar{E}_{\text{ext}}(t) = (E_x \cos(\omega t), 0, E_z \cos(\omega t))$, where E_x and E_z are constants, ω is the angular frequency, and t is time.

The interaction of free charges in the fluid and the electric field results in an additional Lorentz body force term $\rho_e\bar{E}$ in the momentum equation, where charge density $\rho_e = -\varepsilon\nabla^2\psi$ and the total electric field strength $\bar{E} = -\nabla\psi_{\text{tot}}$, ∇ being the vector differential operator. We consider flow (longitudinal, transverse) subjected to a time-periodic axial pressure gradient $-(p_z, p_x)$ and electric field $(E_z\cos(\omega t), E_x\cos(\omega t))$ purely in the z- and x-direction respectively. The longitudinal flow is unidirectional in the z-direction. Its velocity $w(x,y)$ is governed by $\mu\nabla^2 w - \rho\,\partial w/\partial t - p_z + \rho_e E_z\cos(\omega t) = 0$, where $\rho = \mu/v$ is the density of the fluid. The transverse flow is two-dimensional in the (x,y) plane satisfying $\nabla\cdot\bar{u} = 0$ and $\mu\nabla^2\bar{u} - \rho\,\partial\bar{u}/\partial t - (p_x, 0) - \nabla p + \rho_e\bar{E} = 0$, where $p(x,y,t)$ is the locally induced pressure. By virtue of symmetry and anti-symmetry of the problem with respect to the x- and y-axes, the electric potential and the velocity field are solved by eigenfunction expansion where the Fourier coefficients are evaluated numerically by applying point collocation to satisfying the heterogeneous boundary conditions. Further taking average over a unit sectional area, the section-mean velocity and current density can be obtained, constituting the Onsager relations for the fluid and current fluxes.

[a)] Corresponding author. Email: cong@hku.hk

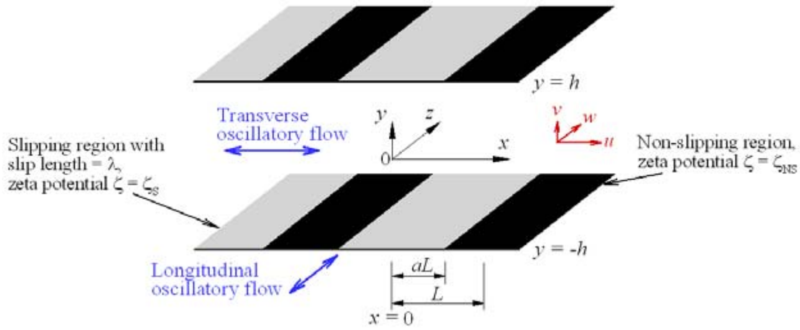

Fig. 1. Electrokinetic longitudinal or transverse flow through a parallel-plate channel with walls which are patterned with a periodic array of stripes.

RESULTS AND DISCUSSIONS

On normalizing lengths by L and potentials by ζ_{NS} (distinguished by hats), let us look into two particular cases. First, when the wall potential is uniform, streaming flow conductance of longitudinal flow $L_{12}^{\parallel}$ and transverse flow $L_{12}^{\perp}$ is:

$$L_{12}^{\parallel,\perp} = \frac{1}{(1-\gamma^2/\hat{\kappa}^2)}\left\{\frac{\tanh(\gamma\hat{h})}{\gamma\hat{h}}\left[\frac{1+\hat{\delta}_{\parallel,\perp}\hat{\kappa}\tanh(\hat{\kappa}\hat{h})}{1+\hat{\delta}_{\parallel,\perp}\gamma\tanh(\gamma\hat{h})}\right]-\frac{\tanh(\hat{\kappa}\hat{h})}{\hat{\kappa}\hat{h}}\right\} \quad \text{for} \quad \hat{\zeta}_S = 1 \tag{1}$$

where $\hat{\delta}_{\parallel,\perp}$ is the effective hydrodynamics slip length of longitudinal and transverse flow which is a complex quantity as discussed by Ng and Wang [6], $\hat{\kappa}$ is the Debye parameter and $\gamma=(1+i)\Omega$ where i is the complex unit and $\Omega = L/\sqrt{2\nu/\omega}$ is an oscillation parameter in which $\sqrt{2\nu/\omega}$ is the Stokes layer thickness. From Eq. (1), the first remark by Squires [1] still holds, as in the steady-flow regime, namely the effective slip length obtained from the hydrodynamic problem can be used directly in the EOF as if the wall were homogeneously slipping. The effect on the streaming conductance by the effective slip length is however nonlinear, unlike the linear effect in the steady case. Second, we consider uncharged perfect-slipping stripes. In the limit of very thin EDL, the streaming conductance is

$$L_{12}^{\parallel,\perp} \approx \left[\frac{1}{1+\hat{\delta}_{\parallel,\perp}\gamma\tanh\left(\gamma\hat{h}\right)}\right]\frac{\tanh(\gamma\hat{h})}{\gamma\hat{h}} \quad \text{for} \quad \hat{\zeta}_S = 0,\ \hat{\lambda}=\infty,\ \hat{\kappa}\hat{h}\gg 1 \tag{2}$$

Clearly from Eq. (2), the EOF is always affected by the effective slip length as long as $\gamma \neq 0$. The second remark by Squires [1], which states that uncharged perfect-slipping regions will give rise to no effect to the EOF for a very thin EDL, is not applicable to the unsteady-flow problem. As shown in Fig. 2, the streaming conductance will indeed tend to unity in the thin-double layer limit for any slipping area fraction $0 \le a < 1$ in the steady-flow regime, i.e., $\Omega = 0$. In the oscillatory regime, for instance $\Omega = 2$, there is however an appreciable decrease in the streaming conductance when a is increased, even under the condition of a very thin EDL. The presence of uncharged stripes, regardless being perfectly slipping, will always have a decreasing effect on the streaming conductance in the oscillatory-flow regime.

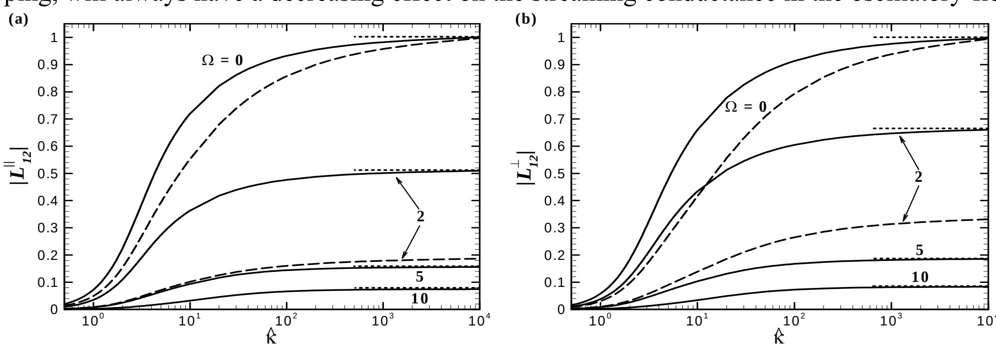

Fig. 2. Magnitude of (a) longitudinal and (b) transverse streaming conductance, $|L_{12}^{\parallel}|$ and $|L_{12}^{\perp}|$, as functions of Debye parameter $\hat{\kappa}$ and oscillation parameter Ω where a = 0.5 (solid) and a = 0.9 (dashed), $\hat{h}=0.5$, $\hat{\lambda}=\infty$, and $\hat{\zeta}_S=0$.

References

[1] Squires T.M.: Electrokinetic Flows over Inhomogeneously Slipping Surfaces. *Phys. Fluids* **20**:092105, 2008.
[2] Bahga S.S., Vinogradova O.I., Bazant M.Z.: Anisotropic Electro-osmotic Flow over Super-hydrophobic Surfaces. *J. Fluid Mech*. **644**:245-255, 2010
[3] Zhao H.: Streaming Potential Generated by a Pressure-driven Flow over Superhydrophobic Stripes. *Phys. Fluids* **23**:022003, 2011
[4] Belyaev A.V., Vinogradova O.I.: Electro-osmosis on Anisotropic Super-hydrophobic Surfaces. *Phys. Rev. Lett.* **107**:098301, 2011
[5] Ng C.O., Chu H.C.W.: Electrokinetic Flows through a Parallel-plate Channel with Slipping Stripes on Walls. *Phys. Fluids* **23**:102002, 2011
[6] Ng C.O., Wang C.Y.: Oscillatory Flow through a Channel with Stick-slip Walls: Complex Navier's Slip Length. ASME *J. Fluids Eng.* **132**:101204, 2010