STABILITY ANALYSIS OF FLOWS IN OPEN CHANNELS PARTIALLY COVERED WITH VEGETATION

Adriano Coutinho de Lima*[a)] & Norihiro Izumi*
*Division of Field Engineering for Environment, Graduate School of Engineering, Hokkaido University, Sapporo, 0608628, Japan

Summary We perform linear and nonlinear stability analyses of flow in open-channels partially covered with vegetation. Instabilities take place in the shear layer at the edge of the vegetated region, and cause the generation of discrete horizontal vortices. We study critical conditions in which these instabilities neither grow nor decay and bifurcation patterns at critical points. In the case of supercritical bifurcation, the amplitude of perturbations can be theoretically determined.

INTRODUCTION

The presence of lateral vegetation in a watercourse causes velocity retardation and creates a sharp lateral gradient in the depth-averaged velocity profile. The eddy viscosity $\tilde{\eta}$ is a parameter necessary to determine the velocity variation along the transverse direction, and is assumed herein as a constant as $\tilde{\eta} = \tilde{U}_{fn}\tilde{D}_n/15$, where $\tilde{U}_f$ is the friction velocity and $\tilde{D}$ is the flow depth (the index n denotes far from the vegetated region). This expression for $\tilde{\eta}$ is theoretically obtained from assuming Prandtl's mixing length theory and a shear stress constant in the vertical direction, near the bed. The obtention process of the eddy viscosity and its inclusion in the governing equations of the flow in a partially vegetated channel have been subject of considerable differences among authors, such as [1], [2] and [5].

FORMULATION

It is assumed that water is flowing through a wide rectangular open-channel which has a lateral zone covered by emerging vegetation (trees), according to Figure 1. The vegetation is approximated into regularly spaced cylinders. The channel has a small constant slope along its streamwise direction.

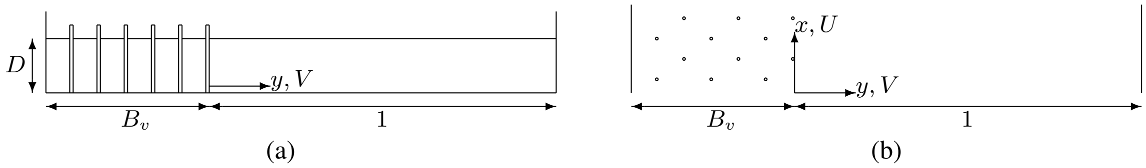

Figure 1. Cross section (a) and upper view (b) of the watercourse, showing dimensionless variables.

Using dimensionless variables, the governing shallow water equations are

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \beta - F_n^{-2}\frac{\partial D}{\partial x} - \beta\left(D^{-1}+\alpha\right)\left(U^2+V^2\right)^{1/2}U + \epsilon\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right) \tag{1}$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -F_n^{-2}\frac{\partial D}{\partial y} - \beta\left(D^{-1}+\alpha\right)\left(U^2+V^2\right)^{1/2}V + \epsilon\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial D}{\partial t} + \frac{\partial UD}{\partial x} + \frac{\partial VD}{\partial y} = 0 \tag{3}$$

where x and y are the coordinates in the streamwise and transverse directions, respectively, t is time, U and V are the velocities in the x and y directions, respectively, D is the flow depth, B_v is the width of the vegetated zone, and β, ϵ, F_n and α are dimensionless parameters: β is strongly correlated to the aspect ratio of the cross section of the flow, ϵ is obtained from the normalization of $\tilde{\eta}$, F_n is the Froude number and α is correlated to the vegetation drag and density. The parameter α assumes the value of zero at the non-vegetated zone and a constant value denoted as α_v at the vegetated zone.

Perturbation expansions

The velocities (U and V) and flow depth (D) are expanded as

$$\begin{aligned} U &= U_0 + \nu(AU_{11}E + cc) + \nu^2(A^2U_{22}E + cc + AA^*U_{20} + U_{00}) + \nu^3(A^3U_{33}E^3 + cc + U_{31}E + cc) \\ V &= \nu(AV_{11}E + cc) + \nu^2(A^2V_{22}E + cc + AA^*V_{20}) + \nu^3(A^3V_{33}E^3 + cc + V_{31}E + cc) \\ D &= 1 + \nu(AD_{11}E + cc) + \nu^2(A^2D_{22}E + cc + AA^*D_{20}) + \nu^3(A^3D_{33}E^3 + cc + D_{31}E + cc), \end{aligned} \tag{4}$$

with $E = e^{i(kx-\omega_r t)+\Omega t}$. The term U_0, which is a function of y, only, is the velocity in the base state, where there is no transverse velocity and the flow depth is equal to 1. The order of the perturbations is expressed by ν. A is the amplitude

a) Corresponding author. E-mail: adriano@eng.hokudai.ac.jp.

of the perturbations, k is their wave number, ω_r is their frequency, Ω is their growth rate, cc is the complex conjugate of the preceding term and A^* is the complex conjugate of the amplitude A. If Ω is positive, the disturbances grow as time progresses, while they decay to vanish if Ω is negative. The terms containing subscripts are functions of y, only. The explicit analytical solution for U_0 is obtained by direct integration of the governing equation in the base state.

LINEAR STABILITY ANALYSIS

From the linear stability analysis, it was determined whether the perturbations would grow or decay for given values of α_v, β, ϵ and F_n (Figure 2). In the linear level, the orders $O(\nu^2)$ and $O(\nu^3)$ are neglected and the corresponding equations were solved using a spectral method with the Chebyshev polynomials. Agreement was verified when comparing the periods of vortex generation predicted herein with the experimental values of [2], [3] and [4].

NONLINEAR STABILITY ANALYSIS

If the amplitude of the perturbations grows to be large, the higher order terms of (4) must be considered. A multiple-scale nonlinear stability analysis with the growth rate expansion method is performed herein. The vicinity of the neutral stability ($\Omega = 0$, where the perturbations nor grow nor decay) is slightly deviated from a critical value of α_v (denoted as α_{vc}) as $\alpha_v = \alpha_{vc} + \nu^2$. The time scale of the amplitude A is ν^{-2}, which is much slower than the time scale of the wavelike part of the disturbance. The slow time scale t_1 is then introduced as $t_0 = t$ and $t_1 = \nu^2 t$. The equations up to the order $O(\nu^3)$ are solved by introducing the slow time scale, the expression for the deviation of α_{vc} and the corresponding critical values of k and ω_r into (1)-(4). Similarly as at the linear level, the Chebyshev polynomials are employed. As a result, the following Landau equation is obtained, where ζ_0 is the linear growth rate and ζ_1 is the first Landau constant.

$$\frac{dA}{dt_1} = \zeta_0 A + \zeta_1 |A|^2 A \tag{5}$$

Supercritical bifurcation occurs when the real part of ζ_0 is positive and the real part of ζ_1 is negative. In such case, the amplitude asymptotically approaches to $\sqrt{-\mathrm{Re}\left(\zeta_0\right)/\mathrm{Re}\left(\zeta_1\right)}$ as $t \to \infty$. It was found that supercritical bifurcation occurs in a wide range of the evaluated parameters (α_{vc}, β, ϵ and F_n), except when the aspect ratio $\tilde{B}/\tilde{D}_n$ is small. When the aspect ratio is sufficiently large, there is a large outer layer with undisturbed velocities which limits the further growth of the disturbances, as in the case presented in Figure 3.

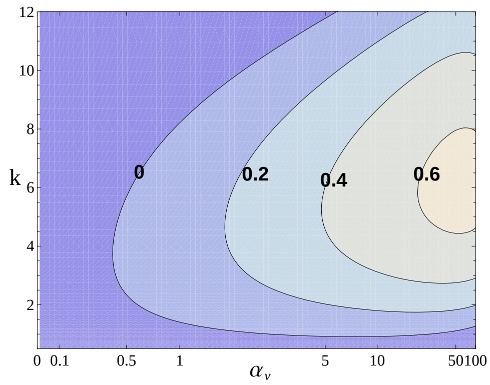

Figure 2. Contours of the growth rate Ω, where $\beta = 0.05$, $\epsilon = 6 \times 10^{-4}$, $B_v = 0.55$, and $F_n = 0.5$.

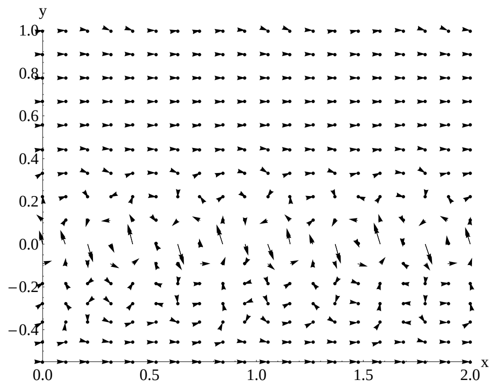

Figure 3. Velocity vectors up to $O(\nu^2)$, where $\beta = 0.10$, $\epsilon = 3 \times 10^{-4}$, $B_v = 0.55$, $F_n = 0.5$, and $\alpha_{vc} = 0.387$.

CONCLUSIONS

The conditions which affect the generation of vortices and the bifurcation pattern in a partially vegetated channel were studied using linear and nonlinear stability analyses. The period of the generated vortices could be theoretically predicted, and in the case of supercritical bifurcation, the scale of the eddies could be determined.

References

[1] Chu V.H., Wu J.H., Khayat R.E.: Stability of Transverse Shear Flows in Shallow Open Channels. *J. of Hydr. Engrg., ASCE* **117**:1370–88, 1991.
[2] Ikeda S., Izumi N., Ito R.: Effects of Pyle Dikes on Flow Retardation and Sediment Transport. *J. of Hydr. Engrg., ASCE* **117**:1459–78, 1991.
[3] Tsujimoto T.: Open Channel Flow with Bank Vegetation. *KHL Communication* **2**:41–54, 1991.
[4] Tsujimoto T., Kitamura T.: Transverse Mixing Associated with Surface Wave in Open-channel Flow with Longitudinal Zone of Vegetation. *Proc. of Hydr. Engrg.* 273–80, 1992.
[5] White B.L., Nepf H.M.: Shear Instability and Coherent Structures in Shallow Flow Adjacent to a Porous Layer. *J. Fluid Mech* **593**:1–32, 2007.