CAN ENERGETICS REALLY CONSTRAIN THE OCEANIC STIRRING BY WIND AND TIDES?

Rémi Tailleux*
*Dept of Meteorology, University of Reading, Earley Gate, PO Box 243, Reading, RG6 6BB, United Kingdom

Summary It has been argued over the past decade that energetics constrains the mechanical power input by the wind and tides to be $O(2TW)$. The argument, however, relies critically on the assumptions that the surface buoyancy fluxes contribute negligibly to the mechanical power input and that the mixing efficiency equals to 20%. This argument is revisited here in the light of new theoretical developments about the nature of buoyancy-driven mixing and mixing efficiency, which reveals many new twists to the issue.

TURBULENT DIAPYCNAL MIXING AND OCEAN CIRCULATION ENERGETICS

Turbulent diapycnal mixing is a key oceanic process that plays a central role in the understanding of ocean/climate interactions, for its intensity is known to have a direct effect on the strength of the Atlantic meridional overturning circulation and its associated northward heat transport. Diapycnal mixing is important for the downward heat transfer from the surface equatorial regions, which is needed for counteracting the cooling of the deep ocean by deep water formation. Physically, diapycnal mixing is ultimately achieved by turbulent molecular diffusion, i.e., the enhanced molecular heat transfer resulting from the stretching of isopycnal surfaces by turbulent mechanical stirring. Such a stretching involves work against buoyancy forces, so that turbulent diapycnal mixing rates requires an external source of power. In the oceans, the main candidates are the wind stress, the surface buoyancy fluxes, and the tides. In an influential study, [3] suggested that energetics considerations could be used to infer constraints on the required magnitude of the mechanical sources of stirring. Thus, by balancing the gravitational potential energy (GPE) loss due to high-latitude cooling (estimated to be $O(0.4\,\mathrm{TW})$) with the GPE increase due to turbulent mixing, estimated as the product of a constant mixing efficiency $\Gamma = 0.2$ times the mechanical power input in the oceans, they argued that the latter should $O(2\,\mathrm{TW})$ in order to balance the observed rate of deep water formation. As the power input due to the wind stress is only $O(1\,\mathrm{TW})$, [3] suggested that only the tides could provide the missing $O(1\,\mathrm{TW})$. The contribution of the surface buoyancy fluxes was dismissed by invoking a popular interpretation of Sandstrom's theorem.

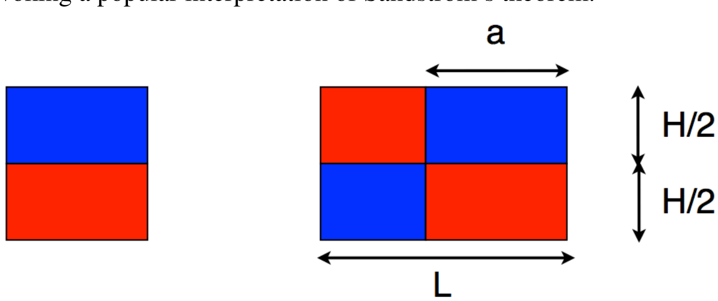

Fig. 1 Classical configuration of the Rayleigh-Taylor instability, with cold fluid (in blue) overlying light (red) fluid (Left panel). Extended configuration where the Rayleigh-Taylor instability is embedded in an otherwise stable stratification (Right panel). The classical Rayleigh-Taylor instability is recovered when $a = L$.

ENERGETICS OF BUOYANCY-DRIVEN MIXING

We seek to assess the validity of [3]'s arguments by seeking a theoretical understanding of the role of surface buoyancy fluxes and of the concept of mixing efficiency. To that end, it is first useful to review the energetics of Rayleigh-Taylor instability, which is widely regarded as the archetypal model of buoyancy-driven mixing. In that case, the energy for the mixing comes from the available potential energy, which in the standard configuration is $AGPE = g(\rho_2 - \rho_1)LH^2/4$. As is well known, irreversible mixing increases the GPE of the background reference state, which is maximised when the system if fully mixed, the maximum GPE_r increase being $\Delta GPE_r = g(\rho_2 - \rho_1)LH^2/8 = AGPE/2$, so that an upper bound for the mixing efficiency is $\Gamma_{max} = \Delta GPE_r/AGPE = (GPE_{r,mixed} - GPE_{r,ini})AGPE = 1/2$.

Case	GPE	GPE_r	$AGPE$	$GPE_{r,mixed}$	Γ_{max}
I	$-\frac{(3\rho_1+\rho_2)LgH^2}{8}$	$-\frac{g(3\rho_2+\rho_1)LH^2}{8}$	$\frac{g\Delta\rho LH^2}{4}$	$-\frac{g(\rho_1+\rho_2)LH^2}{4}$	$\frac{1}{2}$
II	$-\frac{[L(3\rho_2+\rho_1)-2a\Delta\rho]gH^2}{8}$	$-\frac{g(3\rho_2+\rho_1)LH^2}{8}$	$\frac{g\Delta\rho aH^2}{4}$	$-\frac{g(\rho_1+\rho_2)LH^2}{4}$	$\frac{L}{2a}$

Table 1: Values of the different energy quantities associated with the two configurations depicted in Fig. 1. The upper bound for the mixing efficiency is computed as $(GPE_{r,mixed} - GPE_r)/AGPE$, and $\Delta\rho = (\rho_2 - \rho_1)$

Recently, [6] suggested that this relatively low upper bound of mixing efficiency for RT instability was an artefact due to the fact that the standard RT configuration has twice as much available potential energy as is required to achieve complete mixing. As a consequence, half of the initial APE has no choice but to be eventually "wasted" as viscous dissipation. An interesting question is therefore whether this excess APE could actually be used to achieve further mixing, which suggests investigating the mixing efficiency of RT instability embedded in an otherwise stable stratification, as schematically

a) Corresponding author. E-mail: R.G.J.Tailleux@reading.ac.uk

depicted in the right panel of Figure 1 (the left panel depicts the standard configuration of RT instability). It can be shown theoretically that the upper bound for mixing efficiency in that case is $\Gamma = L/(2a)$, where a/L is the relative fraction of the domain where the unstable configuration occurs. Clearly when $a/L = 1$, the classical result is recovered, but one sees that if $a = L/2$, then it is in principle possible to achieve a mixing efficiency of unity. [2] investigated this issue by means of laboratory experiments, and found values of mixing efficiency by embedding Taylor-Taylor instability in an otherwise stable stratification significantly higher than the maximum upper bound $\Gamma = 1/2$ for standard Rayleigh-Taylor instability. Motivated by these ideas, [6] and [7] sought to understand the behaviour of the mixing efficiency in the context of horizontal convection, with and without an external source of mechanical stirring. In absence of mechanical forcing, the main result is that the mixing efficiency increases as the Rayleigh number R_a increases. Based on all current theoretical evidence, reviewed by [5], the expectation is that the mixing efficiency varies as:

$$\Gamma = 1 - R_a^{-\alpha}, \qquad 0 < \alpha \leq 1/3, \tag{1}$$

and hence that the mixing efficiency converges asymptotically towards unity in the limit $R_a \to +\infty$, indicating perfect efficiency. In such an approach, the mixing efficiency is defined as $\Gamma = D(APE)/[D(APE)+D(KE)]$, where $D(APE)$ and $D(KE)$ are the volume-integrated diffusive dissipation of APE and viscous dissipation of KE respectively, where the APE dissipation is defined as in [8]. In a specific example, [5] find in a numerical experiment of horizontal convection at $R_a = 10^{10}$ a value of mixing efficiency slightly above 0.85, which is significantly larger than all published values of buoyancy-driven mixing efficiency, and considerably larger than the canonical value $\Gamma = 0.2 - 0.25$ widely used by oceanographers, and typical of mechanically-driven mixing. [5] concluded that horizontal convection is very efficient at achieving very high mixing with relatively low levels of viscous dissipation. In contrast, the addition of mechanical stirring to horizontal convection was found to systematically *decrease* the overall mixing efficiency in [7]. In particular, values closer to the canonical value $\Gamma \approx 0.2$ could be achieved only when the mechanical power input due to the mechanical forcing was significantly higher than the APE production rate by the surface buoyancy fluxes.

CONCLUSIONS

Until recently, buoyancy-driven turbulent mixing had been assumed to be only marginally more efficient than mechanically-driven mixing, based on laboratory measurements of mixing efficiency of Rayleigh-Taylor instability being only slightly above the canonical value $\Gamma = 0.2 - 0.25$ often reported for mechanically-driven mixing. Theoretical considerations, however, strongly suggest that the relatively low values of mixing efficiency found in Rayleigh-Taylor instability are due to the fact that there is twice as much initial APE as needed to fully mix the initially unstable stratification, which could in principle be used to achieve further mixing. As a result, embedding the Rayleigh-Taylor instability in an otherwise stable stratification potentially yields larger value of mixing efficiency, which was confirmed by [2] by means of laboratory experiments, and by [7, 5] in the context of numerical experiments of horizontal convection. Recently, [6, 1] provided new theoretical support for the idea that the surface buoyancy fluxes contribute significantly to the mechanical power input in the oceans (in agreement with [4], who estimated $G(APE) = 1.2 \pm 0.7\text{PW}$), in contrast to the claims made by [3]. The main consequence is that the surface buoyancy fluxes affect the energetics of mixing by modulating the overall value of the bulk mixing efficiency Γ required to convert the loss of GPE due to cooling into the mechanical power input due to the wind and tides. As shown by [7], even if buoyancy-driven mixing is relevant only in a few localised places in the oceans, it may be sufficiently important to double the overall value of mixing efficiency. As a result, [3] is questionable, because the present evidence is that the surface buoyancy fluxes contribute significantly to the problem, and that the relevant value of mixing efficiency must be determined as part of the problem, and hence cannot be assumed to be $\Gamma \approx 0.2$. As a result, it does not appear possible to constrain the sources of mechanical stirring, because not enough is known yet about the theoretical value of mixing efficiency achieved in a system where the mixing is both buoyancy- and mechanically-driven.

References

[1] Hughes, G. O., Hogg, A., Griffiths, R.W.: Available potential energy and irreversible mixing in the mieridional overturning circulation. *J. Phys. Oceanogr.* **39**:3130–3146, 2009.

[2] Lawrie, A., G. W., Dalziel, S. B.: Rayleigh-Taylor mixing in an otherwise stable stratification. *J. Fluid Mech.* **688**:507–527, 2011

[3] Munk, W., Wunsch, C.: Abyssal recipes II: Energetics of tidal and wind mixing. *Deep Sea Res.* **45**:1977–2010, 1998.

[4] Oort, A. H., Anderson, L. A., Peixoto, J. P.: Estimates of the energy cycle of the oceans. *J. Geophys. Res.* **99**:7665–7688.

[5] Scotti, A., White, B.: Is horizontal convection really "non-turbulent"? *Geophys. Res. Lett.* **38**, L21609, 2011.

[6] Tailleux, R.: On the energetics of stratified turbulent mixing, irreversible thermodynamics, Boussinesq models, and the ocean heat engine controversy. *J. Fluid Mech.* **632**:339–382, 2009

[7] Tailleux, R., Rouleau, L.: The effect of mechanical stirring on horizontal convection. *Tellus* **62A**:138–153, 2010.

[8] Winters, K.B., Lombard, P., Riley, J. J., d'Asaro, E.: Available potential energy and mixing in density-stratified fluids. *J. Fluid Mech.* **289**:115-228, 1995.