THE EVOLUTION AND STABILITY OF AXISYMMETRIC INTRUSIONS

Bruce R. Sutherland*[a], Colm-cille P. Caulfield** and Justine M. McMillan***
*Departments of Physics and of Earth and Atmospheric Sciences, University of Alberta, Edmonton, Alberta, T6G 2E1, Canada
**BP Institute and the Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, CB3 0EZ, United Kingdom
***Department of Physics, University of Alberta, Edmonton, Alberta, T6G 2E1, Canada

Summary Radially spreading intrusions in a two-layer fluid behave qualitatively differently than their bottom-propagating gravity current counterparts. The intrusion fronts become azimuthally asymmetric and, rather than decelerate, they propagate at constant speed for long distances. These dynamics are examined through extension of the Korteweg de Vries (KdV) equation to cylindrical geometries and through the use of optimal modes.

BACKGROUND

A gravity current usually refers to the flow over a bottom boundary of dense fluid into a less dense ambient[6]. The flow is driven by horizontal pressure gradients resulting from the relative buoyancy force between the current and ambient. In lock-release experiments performed in rectangular tanks, a constant volume of dense fluid quickly reaches a steady state speed until propagating approximately 6 to 10 lock lengths at which point the current head becomes depleted of fluid and the flow decelerates as the head height, and hence the horizontal pressure gradient, decreases[4]. If a constant volume is released from a cylindrical lock, the gravity current spreads radially as an expanding annulus. In this case the flow decelerates because the head height must decrease as the circumference of the annulus increases[1].

If a gravity current propagates into a stratified ambient, its evolution can be significantly more complex due to the generation of internal waves formed by the collapsing fluid in the lock and due to the motion of the head itself. In the simple circumstance in which the ambient is a two-layer fluid with equal upper- and lower-layer depths and the density of the lock-fluid is the average ambient density, symmetry dictates that the intrusion should move along the interface identically to a gravity current moving over a free-slip bottom into a uniform ambient of half the depth. But experiments have shown this is not the case. In a cylindrical geometry the intrusion is found to propagate at constant speed far from the lock[2], and the structure of the radially spreading intrusion head is less axisymmetric than that of a gravity current[5] (see Figure 1).

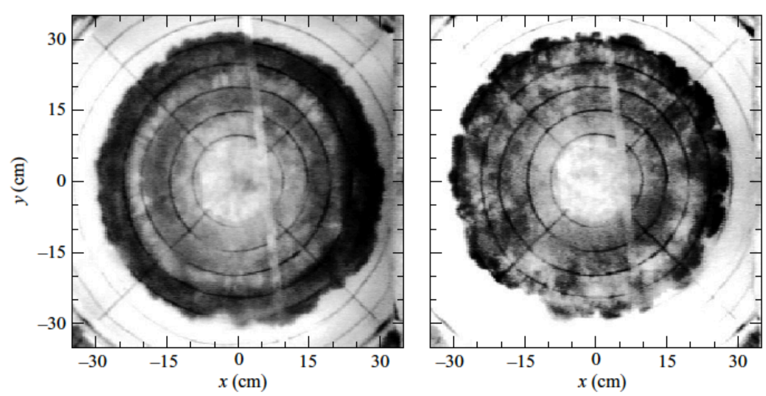

Figure 1. Top view snapshots from laboratory experiments showing (left) a bottom-propagating gravity current and (right) an intrusion at the interface of a two-layer fluid. In both experiments the dyed lock-fluid was released from a cylinder of radius 6 cm in fluid of total depth $H = 10\,\mathrm{cm}$.

The reason these experiments defy expectations is that the interface between two miscible fluids cannot be treated as infinitesimally thin. When the intrusion leaves the lock it launches a mode-2 (varicose) interfacial wave which itself qualitatively changes the evolution of the intrusion. Mathematical tools are adapted here for the purpose of examining these effects.

SIMULATIONS AND THEORY

Fully nonlinear numerical simulations using parameters of the laboratory experiments reproduced the observed speed of the intrusion. To see whether azimuthal asymmetries played a role in establishing the intrusion speed and propagation distance, the simulations assumed the flow was axisymmetric.

[a] Corresponding author. E-mail: bruce.sutherland@ualberta.ca.

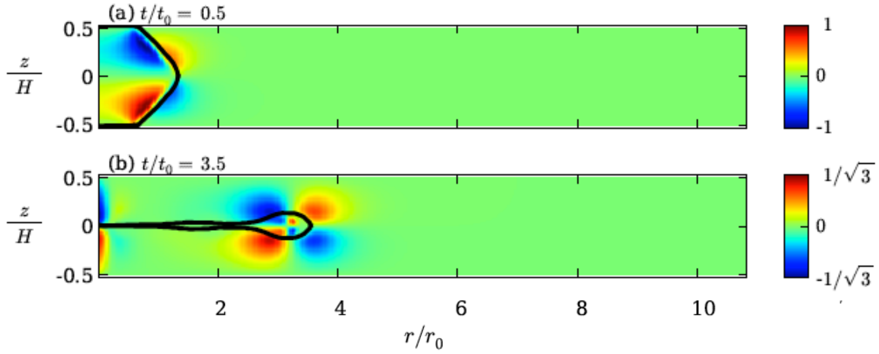

Figure 2. Simulation of an axisymmetric intrusion at two times showing vertical velocity (colour) and the position of the intrusion front (black line) as a function of radius and height. Here H the total fluid depth, r_0 is the lock-radius and t_0 is the predicted time to travel one lock-radius. The vertical velocity field is scaled to account for the predicted decrease as $1/\sqrt{r} \propto 1/\sqrt{t}$.

Figure 2 shows vertical-radial cross-sections of the flow at two times. Shortly after release, the generation of a mode-2 internal wave was evident through the vertical velocity field. This then surrounded the head and propagated radially at constant speed transporting the head with it.

Following the approach of Miles[3], an evolution equation for the amplitude of a cylindrical solitary wave was derived. Assuming the vertical displacement field is separable such that $\xi(r, z, t) = a(r, t)\phi(z)$, the amplitude evolution equation analogous to the KdV equation is

$$a_t + c_0(a_r + a/2r) + \gamma(aa_r + a^2/2r) + \beta a_{rrr} = 0, \tag{1}$$

in which the constants c_0, γ and β depend upon the vertical structure function $\phi(z)$. As with cylindrical solitary surface waves[7], the derivation of this equation is flawed in that it necessarily assumes r is large; this removed the balance between weak nonlinearity and dispersion required of standard KdV theory.

Indeed, examination of the simulations show that the amplitude conforms well to a scaled KdV solution of the form

$$a(r, t) \propto r^{-1/2}\text{sech}^2[(x - ct)/\lambda],$$

in which c and λ depend upon the magnitude of the amplitude, a_0. for which the corresponding partial differential equation is

$$a_t + c_0(a_r + a/2r) + \gamma r^{1/2}(aa_r + a^2/2r) + \beta a_{rrr} = 0, \tag{2}$$

The introduction of $r^{1/2}$ to the nonlinear terms suggest a different balance from that typically applied in KdV theory.

Although the intrusion speed and stopping distance were well captured by simulations restricted to an axisymmetric geometry, experiments also showed that an intrusion emanated from the lock not as an expanding annulus but as a series of spokes. A mathematical understanding this azimuthal symmetry breaking is not straightforward because standard normal-mode stability analyses assume a basic state that is steady or slowly evolving compared to the instability time-scale. Experiments reveal this is not the case for axisymmetric intrusions. Instead we analyze the azimuthal stability of the intrusion through the method of optimal modes applied to the time-dependent basic state of the collapsing lock-fluid.

CONCLUSIONS

The evolution of intrusions and waves in a cylindrical geometry provide new mathematical challenges. We have shown that it is necessary to blend insights from laboratory experiments and simulations in order to gain insights into the appropriate development of theories for the nonlinear wave evolution and azimuthal intrusion structure.

References

[1] H. E. Huppert and J. E. Simpson. The slumping of gravity currents. *J. Fluid Mech.*, 99:785–799, 1980.
[2] J. M. McMillan and B. R. Sutherland. The lifecycle of axisymmetric internal solitary waves. *Nonlin. Proc. Geophys.*, 17:443–453, 2010.
[3] J. W. Miles. An axisymmetric boussinesq wave. *J. Fluid Mech.*, 84:181–191, 1978.
[4] J. W. Rottman and J. E. Simpson. Gravity currents produced by instantaneous releases of a heavy fluid in a rectangular channel. *J. Fluid Mech.*, 135:95–110, 1983.
[5] B. R. Sutherland and J. T. Nault. Intrusive gravity currents propagating along thin and thick interfaces. *J. Fluid Mech.*, 586:109–118, 2007. doi:10.1017/S0022112007007288.
[6] M. Ungarish. *An Introduction to Gravity Currents and Intrusions*. Chapman and Hall/CRC press, New York, 2009.
[7] P. D. Weidman and R. Zakhem. Cylindrical solitary waves. *J. Fluid Mech.*, 191:557–573, 1988.