CAPILLARY RISE IN AN ANNULAR CONICAL SPACE

A Medina*, C. A. Vargas* & A. López-Villa**
* *Laboratorio de Sistemas Complejos, Departamento de Ciencias Básicas, UAM Azcapotzalco, Av. San Pablo 180, C. P. 02250 México D.F., México.*
** *SEPI ESIME Azcapotzalco, Instituto Politécnico Nacional, Av. de las Granjas 682, Col. Santa Catarina, 02250, Azcapotzalco D.F., México*

Summary In this work we analyze theoretically, using the lubrication theory, the spontaneous capillary rise under the gravity field, of a viscous liquid into the gap made by two coaxial hollow cones. As a result we can conclude that in this case the time evolution of the front of liquid is initially slow and after, for a long period is near constant and finally it is accelerated very strongly. This behavior is atypical in processes of capillary rise.

INTRODUCTION

Spontaneous capillary rise of viscous liquids into pipes is a fundamental phenomenon [1-6] that allows understanding uptake of liquids in plants [7], paper [8-10] and other porous media like oil reservoirs [11]. Capillary rise also occurs in very narrow structures like nano channels [12,13] and in structurally very complex gaps [14,15]. A special type of axisymmetric capillary penetration appears in horizontal radial Hele-Shaw cells [4,16], there the liquid spontaneously penetrates the cell without the influence of gravity. Despite it the capillary penetration first is slow and at final stages it is slightly accelerated [4].

In this work we consider an axisymmetric configuration, a conical Hele-Shaw cell, where the gravity field can modify the spontaneous penetration. In this capillary structure the changes in geometry and the gravity acceleration competes. The conical Hele-Shaw cell is made of two concentric cones having a small gap between them. In this case the front of the liquid that penetrates due to the action of the capillary pressure has a smooth, uniform profile which in a first approximation does not depend on the azimuthal coordinate θ. Because of the flow in the gap is slow and very slim it can be seen as a film flow and it is possible to use the lubrication theory to model the behavior of this flow. The main result obtained from this analysis is that the capillary penetration initially is slow, after it moves near a constant speed and very slowly and, finally, it is suddenly accelerated very strongly.

CAPILLARY RISE IN THE GAP BETWEEN CONES

The capillary rise occurs when a viscous liquid, of dynamic viscosity μ and density ρ is in contact with a capillary space made of a wetting material. There the capillary pressure, p_c, forces the liquid to fill the space, even if the gravity field is present. In Fig. 1 we show schematically the flow developed in the gap between two concentric cones. Both hollow cones have the same opening angle α and make a gap of uniform size *h*. It is assumed that $h \ll (L, l_c)$, where L is the height of the cone and l_c is the capillary length ($l_c = \sqrt{\sigma/\rho g}$, here σ is the liquid's surface tension and *g* is the gravity acceleration [4]). In general, l_c is of the order of a few mm.

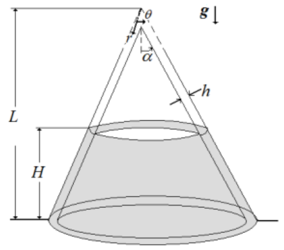

Fig. 1

Thus, this flow developed in the gap can be described by using the lubrication equations in polar coordinates (r,θ). The mass conservation can be given in terms of the Reynolds equation in polar coordinates, which is an equation for the lubricant pressure P [18-20], it takes the form

$$\frac{\partial}{\partial r}\left(\frac{h^3}{12\mu} r \sin\alpha \frac{\partial P}{\partial r}\right) = 0, \tag{1}$$

and the corresponding volume flow, per unit length, along the r -direction, q_r, is now

$$q_r = -\frac{h^3}{12\mu}\frac{\partial P}{\partial r}. \tag{2}$$

Equations (1) and (2) must be solved under the following boundary conditions

$$\text{at } r = \frac{L}{\cos\alpha},\ P = 0, \quad \text{at } r = \frac{L-H}{\cos\alpha},\ P = \frac{2\sigma\cos\theta_c}{h} - \rho g(L-H). \tag{3}$$

In (3) H is the position of the liquid front and in the equation for pressure the first term is the capillary pressure valid when (L-H)>>h, finally, there θ_c is the angle of contact. Developing and integrating we obtain the equation

$$\int_0^H \frac{\ln\left(\frac{L-H}{L}\right)}{\rho g - \frac{2\sigma\cos\theta_c}{h(L-H)}} dH = \frac{h^2 \cos\alpha}{12\mu} t. \tag{4}$$

RESULTS

The numerical integration of Eq. (4) requires to give particular values to the physical quantities corresponding to liquids of interest. We assume ambient conditions and a conical Hele-Shaw cell with $L=0.1$m height, $\alpha=30^o$ aperture angle, and $h=0.1$ mm gap size, which can be penetrated by water and 50 cP and 100 cP silicon oils, in computations we assume, without lost of generality, a perfect wetting, i.e., $\theta_c=0^o$. At room conditions (temperature $20\ ^\circ$C and atmospheric pressure) water has the following properties:$\rho=998$ kg/m^3, σ=0.072N/m, μ=1.002cP, meanwhile Silicon oil 50 cP has ρ=960 kg/m^3, σ=0.02086 N/m, μ=50 cP and Silicon oil 100cP has the values ρ=962 kg/m^3, σ=0.02086 N/m and μ=100 cP. In Fig. 2 we show graphically the main results: initially the capillary flow is very strong (high slope), after the slope remains constant by around 2/3 of the time of rise and finally the flow once again it is accelerated.

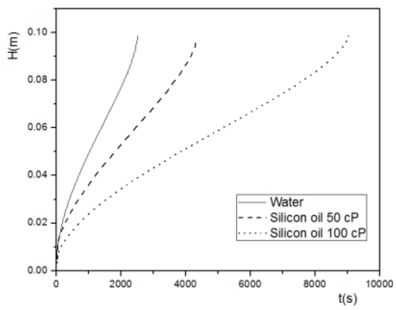

Fig. 2

CONCLUSIONS

To our knowledge, this is the first time that capillary rise is analyzed in this geometry. In this work this problem was treated by using the lubrication theory. The numerical solutions of the motion equation for several liquids of interest allowed us to found that in all cases the capillary rise is very slow initially, and after, there are wide ranges of constant speeds, mainly for cases of very viscous liquids; finally, at the final stages, sudden increases in the speed of the capillary rise were estimated. This type of behavior is atypical in the processes of capillary rise and a more detailed study of such results is necessary, mainly experimentally. Work along this line is in process.

References

[1] Lucas R.: *Kolloid-Zeitschrift*, **23**:15 1918.
[2] Bell J. M., Cameron F. K.: *J. Phys. Chem.* **10** 658, 1906.
[3] Washburn E.W.: *Phys. Rev.* **17** 273, 1921.
[4] Middleman S.: Modeling Axisymmetric Flows, Academic Press, New York, 1995.
[5] Zhmud B. V., Tiberg F., Hallstensson K.: *Jour. Colloid Interface Sci.* **228** 263, 2000.
[6] de Gennes P-G., Brochard-Wyart, F., Queré D.: Capillarity and wetting phenomena, drops, bubbles, pearls, waves. Springer-Verlag, Berlin, 2004.
[7] Zimmerman M. H., Xylem Structure and the Ascent of Sap, Springer-Verlag, Berlin, 1983.
[8] Medina A., Pérez-Rosales C., Pineda A. and Higuera F. J., *Rev. Mex. Fis.* **47** 537, 2001.
[9] Alava M., Niskanen K., *Rep. Prog. Phys.* **69** 669, 2006.
[10] Li, X. Tian, J. Shen W.: *Cellulose* **17** 649, 2010.
[11] T. Babadagli, *Jour. Pet. Sci. Eng.* **14** 197, 1996.
[12] A. Han, G. Mondin, N. G. Hegelbach, N. F. de Rooij, U. Staufer, *Jour. Colloid Interface Sci.* **293** 151, 2006.
[13] M. Pliego, G. J. Gutiérrez and A. Medina, *Rev. Mex. Fis.* **57** 1, 2011.
[14] F. J. Higuera, A. Medina and A. Liñán, *Phys. Fluids* **20** 102102, 2008.
[15] Gutiérrez G.J., Medina A., F.J. Higuera, *Jour. Colloid Interface Sci.* **338** 519, 2009.
[16] Schutt H., Spetzler H.: *Pure Applied Geophysics* **158** 627, 2001.
[17] Miranda J. A.: *Phys. Rev. E* **65** 036310, 2002.
[18] O. Reynolds, *Philos.Trans. Roy. Soc. A* **177** 157. 1886.
[19] Szeri A. Z.: Fluid film lubrication, Cambridge University Press, Cambridge, 1998.
[20] Liñan A., Rodríguez M., Higuera F.J.: Mecánica de Fluidos (Primera Parte), Escuela Técnica Superior de Ingenieros Aeronáuticos, Madrid, 2003.