LINEAR STABILITY ANALYSIS OF THIN LIQUID FILM CONTAINING SOLUBLE SURFACTANT HEATED UNIFORMALY

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Summary The flow of a thin liquid film containing soluble surfactant is considered in the present paper. Three coupled nonlinear equations for the film thickness, surface and bulk surfactant concentrations are derived based on lubrication approximation. Linear stability is investigated in detail for evolution equations with normal mode method. Results are presented and analyzed according to the effects on stability of parameters of solubility number, bulk Peclect number and surface Biot number.

INTRODUCTION

The thin film flow containing surfactants or nanoparticales has attracted considerable interest in the literature, both theoretically and experimentally. Such flows that are driven by so-called interfacial Marangoni stresses are of great importance in a wide range of industrial and biomedical applications [1-5].

This kind of flow is influenced by many parameters, such as surfactant solubility, concentration and thermocapillary at the interfacial. Lots of experiment studies show that there formed a moving circular wave in the drop centre and the time evolution of the front motion substantially depends on the surfactant solubility [3]. In the theoretical research, Matar studied the spreading of soluble surfactant by a series of work and effects of system parameters to the stability were obtained by normal mode method [4] and transient growth method, respectively [1,5].

When the presence of non-uniform interfacial temperature is considered, thin liquid films are subjected to gradients in surface tension, or thermocapillary, and that will lead to Marangoni flow and noticeable deformation of the liquid film. The flow of thin-film layer may be driven to rupture, which is undesirable for most applications. Most of previous research work focused merely on the influence of the temperature or surfactant concentration. In both thermal and surfactant-driven Marangoni cases, the literature is few. Edmonstone [2] investigated a thin liquid film covered with an insoluble surfactant monolayer of initially constant concentration and subjected to interface heating. Models describing the simultaneous effects of thermal and soluble surfactant-driven flows have not been developed so far.

Following the work of Edmonstone et al.[2], the influence of soluble surfactant on the linear stability of surfactant-covered films flow is investigated in present paper. The evolution equations evolving film thickness, interface and bulk surfactant concentrations considering thermal effect are presented. The linear stability is analyzed using normal mode method and effects of several parameters on the flow stability are examined.

MATHEMATICAL FORMULATION

We consider a thin film of soluble surfactant solutions lying on a horizontal plane. The film is incompressible Newtonian fluid and has initial uniform thickness. The film is bounded from below by a rigid, no slip, impermeable solid substrate and from above by an essentially inviscid gas. The film is heated from below uniformly. Both the viscosity and density of the fluid are assumed to be constant. The surface tension of the fluid is lowered from its maximal value by the presence of surfactant and temperature increase caused by the interface heating.

Assuming that there is no phase transformation at the air-liquid interface, the coupled nonlinear dimensionless evolution equations of film thickness, surface and bulk concentrations can be derived by lubrication theory and cross-sectional averaging method, which are as following:

$$h_t = \left[-Ch^3 h_{xxx} / 3 + 0.5(1-\Sigma) h^2 \Gamma_x - 0.5 \Sigma h^2 B_h h_x / \left(1 + B_h h\right)^2 \right]_x \tag{1}$$

$$\Gamma_t = \frac{\Gamma_{xx}}{Pe} - \left[0.5Ch^2 h_{xxx} \Gamma - \left(1-\Sigma\right) h \Gamma \Gamma_x + \Sigma h \Gamma B_h h_x / \left(1 + B_h h\right)^2 \right]_x + K_s \left(c - \Gamma\right) \tag{2}$$

$$c_t = \left(h_x c_x + h c_{xx}\right) / \left(h Pe_b\right) - \left[Ch^2 h_{xxx} / 3 - 0.5h\left(1-\Sigma\right)\Gamma_x + 0.5h \Sigma B_h h_x / \left(1 + B_h h\right)^2 \right] c_x - \beta K_s \left(c - \Gamma\right) / h \tag{3}$$

Where, C is capillary number, β is the solubility parameter of surfactant, Σ is a Marangoni parameter relating to surface tension, K_s provides a measure of sorption kinetics, Pe and Pe_b are Peclet numbers measuring the ratio of surfactant transport by Marangoni stresses to diffusion along the interface and in the bulk, respectively, B_h is the surface Biot number.

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LINEAR STABILITY ANALYSIS

We study the linear stability of Eqs. (1) to (3) in order to determine which systems are vulnerable to applied perturbations. Moreover, the linear theory growth rate predictions are base of further numerical results which can serve as an important check on the solution methods. Using normal modes, we decompose $h(x, t)$ and $\Gamma(x, t)$, $c(x, t)$ as follows,

$$h = h_0(x,t) + \widetilde{H}e^{ikx}e^{\omega t},\ \Gamma = \Gamma_0(x,t) + \widetilde{\Gamma}e^{ikx}e^{\omega t},\ c = c_0(x,t) + \tilde{c}e^{ikx}e^{\omega t} \tag{4}$$

in which $h_0(x, t)$, $\Gamma_0(x, t)$ and $c_0(x, t)$ represent base states, while $\widetilde{H}$, $\widetilde{\Gamma}$ and $\tilde{c}$ are the amplitudes of initially small periodic perturbations of wavenumber, k, and (complex) growth rate, ω.

Substitution of Eq. (4) into the evolution equations for h, Γ and c and subsequent linearization about $h_0 = 1$ and $\Gamma_0 = c_0 = \gamma_s$ yield the following dispersion equation for perturbation with the growth rate ω and wave number k:

$$\omega^3 + \left(A_2k^4 + A_1k^2 + A_0\right)\omega^2 + (E_2k^6 + E_1k^4 + E_0k^2)\omega + (F_2k^8 + F_1k^6 + F_0k^4) = 0 \tag{5}$$

where

$$A_2 = \frac{1}{3}C,\ A_1 = Pe_b^{-1} + Pe^{-1} + (1-\Sigma)\gamma_s - 0.5B_h\Sigma/(1+B_h),\quad A_0 = K_s(\beta-1),\ E_2 = C(1-\Sigma)\gamma_s/12 + C(Pe_b^{-1} + Pe^{-1})/3,$$

$$E_1 = CK_s(\beta-1)/3 + Pe_b^{-1}\gamma_s(1-\Sigma) + Pe_b^{-1}Pe^{-1} + 0.5B_h\gamma_s\Sigma(1-\Sigma)/(1+B_h)^2 - 0.5B_h\Sigma(\gamma_s(1-\Sigma) + Pe_b^{-1} + Pe^{-1})/(1+B_h)$$

$$E_0 = \beta\gamma_sK_s(1-\Sigma) + K_s(Pe_s^{-1}\beta - Pe_b^{-1}) - 0.5B_hK_s\Sigma(\beta-1)/(1+B_h),\ F_2 = Pe_b^{-1}C\gamma_s(1-\Sigma)/12 + CPe_s^{-1}Pe_b^{-1}/3,$$

$$F_1 = C\gamma_sK_s\beta(1-\Sigma)/12 + CK_s(Pe_s^{-1}\beta - Pe_b^{-1})/3 + 0.5Pe_b^{-1}B_h\gamma_s\Sigma(1-\Sigma)/(1+B_h)^2$$

$$-0.5Pe_b^{-1}B_h\gamma_s\Sigma(1-\Sigma)/(1+B_h) - 0.5Pe_b^{-1}Pe_s^{-1}B_h\Sigma/(1+B_h)$$

$$F_0 = +0.5\beta B_h\gamma_sK_s\Sigma(1-\Sigma)/(1+B_h)^2 - 0.5\beta B_h\gamma_sK_s\Sigma(1-\Sigma)/(1+B_h) - 0.5B_hK_s\Sigma(Pe_s^{-1}\beta - Pe_b^{-1})/(1+B_h)$$

From Eq.(5) it can be seen that both the growth rate ω and the critical wavenumber k_c are dependent of all the system parameters, which are capillary number C, Marangoni parameter Σ, initial surfactant concentration γ_s, Peclect number Pe and Pe_b, surface Biot number B_h and solubility number β. The effects of former four parameters in the model of present paper are identical with insoluble model of Edmonstone [2], so the effect of the latter three system parameters relating to soluble surfactant will be studied intensively.

Eq.(5) is solved firstly to find the critical wavenumber k_c then to find growth rate for a variety of system parameters. Typical value of the parameters used for calculation is chosen from the range in [1-2, 5] . Results show that k_c increases with surface Biot number B_h and decreases with bulk Peclect number Pe_b and solubility number β, indicating that B_h play destabilizing role, while Pe_b and β play stabilizing role .

The dispersion curves reveals that the system becomes more unstable following an increase in B_h and a decrease in β. Similar observations about the effect of β was made in [4-5]. Among the four parameters, the increase of B_h leads to the most remarkable increment in the maximal growth rate. An increase in B_h corresponds to higher thermal resistance of inner heat conductivity or lower heat resistance of outer heat convection. Thus larger Biot number leads to larger temperature gradients across the film height, hence increasing thermocapillary. The effect of B_h is different from the results predicted by the insoluble model by Edmonstone [2]. In the insoluble case, a most dangerous situation was encountered at a value of $B_h = 1$, moving away from this value in either direction appeared to give rise to relatively more stable situations. The maximal growth rate turns larger with Pe_b increases but a shift of k_m and k_c towards small k values. So Pe_b number plays destabilizing role, similar to the results made by transient growth analysis from a no thermocapillary model by WCM [1].

CONCLUSIONS

We have considered the stability of liquid film covered by soluble surfactant spreading on the solid substrate with interface heating. Lubrication theory was used to derive a coupled set of one dimension nonlinear evolution equations for the film thickness and surface and bulk surfactant concentrations. Linear stability analysis was conducted by normal mode method. Results indicate that both Pe_b and B_h play destabilizing roles and β plays stabilizing roles. The most important parameter is the surface Biot number B_h that it leads to most remarkable increment in the maximal growth rate to destabilizing the flow.

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