RIVULET EVOLUTION IN GRAVITY-DRIVEN THIN-FILM FLOWS

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Summary The problem of rivulet formation in the case of thin film flow down an inclined rigid substrate is investigated numerically and comparisons drawn with flow visualisations obtained experimentally. Three dimensional results are presented for the flow of both fully and partially wetting fluids and relationships for the emerging width and wavelength of the rivulet pattern are found. The wetting properties of the fluid are seen to have a significant impact on the characteristics of the emerging instability.

INTRODUCTION

The motion of a thin film down an inclined planar substrate can result in complex behaviour and interesting dynamics at the associated advancing front which becomes unstable, forming a periodic pattern of finger shaped rivulets. Huppert [1] presented the first detailed study of the problem, showing the critical wavelength of the emerging instability, when scaled with the capillary length of the fluids considered, to be captured by a single linear fit of the data. Subsequent theoretical investigations have concentrated, in the main, on fully wetting fluids and their associated linear stability of the travelling wave solution [2]. An alternative approach is to explore the full three-dimensional problem, requiring a numerical solution of the associated governing equations. This is the route adopted below, based on the simplifying assumption that the flow can be considered lubrication like [3].

PROBLEM FORMULATION AND METHOD OF SOLUTION

The problem of interest consists of a thin fluid film, of asymptotic thickness H_0, flowing down a substrate inclined at an angle θ to the horizontal, with a volumetric flow rate Q_0 per unit width. The fluid is considered to be incompressible with constant density, ρ, viscosity, μ, and surface tension, σ, and the film to be fully or partial wetting. Under the assumption that H_0 is small compared to the capillary length, $L_0 = H_0/(6Ca)^{\frac{1}{3}}$, where $Ca\ O(\epsilon^3) << 1$ is the capillary number (= $\mu U_0/\sigma$), that is $H_0/L_0 << 1$, taking $U_0 = 3Q_0/2H_0$ and adopting appropriate scaling, [3], the governing Navier-Stokes and continuity equations reduce to a coupled system of partial differential equations for the film thickness, h, and pressure, p:

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left[\frac{h^3}{3}\left(\frac{\partial p}{\partial x} - 2\right)\right] + \frac{\partial}{\partial y}\left[\frac{h^3}{3}\left(\frac{\partial p}{\partial y}\right)\right], \quad (1)$$

$$p = -\frac{\epsilon^3}{Ca}\nabla^2(h) + 2\epsilon\,(h - z)\cot\theta - \Pi(h), \quad (2)$$

where $\Pi(h)$ is a disjoining pressure term, [4], that incorporates the equilibrium contact angle, θ_c, of the form

$$\Pi(h) = \frac{\epsilon\,(n-1)\,(m-1)\,(1-\cos\theta_c)}{Ca\,(n-m)\,h^*}\left[\left(\frac{h^*}{h}\right)^n - \left(\frac{h^*}{h}\right)^m\right]. \quad (3)$$

If the film is fully wettting, i.e. $\theta_c = 0^o$, then $\Pi(h) = 0$.
Flow upstream is considered to be fully developed where the film thickness H_0 is fixed, the remaining boundary conditions which close the problem are as prescribed in [6]; initially, at time zero, the front itself is perturbed with a superposition of N modes with random length, l_j, and differing wavelength, $\lambda_{0,j}$, as in [5] via $h(x,y) = 0.5\{1 + h^* - (1-h^*)\tanh[x - x_f(y)]\}$; $x_f(y) = x_u - \sum_{j=1}^{N} L_r \cos 2\pi y/\lambda_{0,j}$, where x_u is the position of the unperturbed front.
Despite the simplifying assumptions made, the associated computational challenges are significant given the extent of the solution domain involved and the long-time solutions required, the latter being the first of their kind; the case of partially wetting films further adds to computational effort required. Accordingly, a key feature of the methodolgy adopted to solve the discretised governing equations accurately is one based on a strategy employing automatic error controlled adaptive time stepping and mesh refinement/derefinement within an efficient multigrid framework; further details are available in [6].

RESULTS

Figure 1 provides typical examples of the types of flow which evolve, showing snap-shots of the formation and evolution of rivulets at the advancing front of a fluid film having density, $\rho = 1075$ kg m^{-3}, surface tension, $\sigma = 0.069$ Pa m and viscosity, $\mu = 3.11 \times 10^{-2}$ Pa s (Fluid A as used in the experiments of Johnson *et al.*[7]) flowing down a substrate inclined at 85^o to the horizontal. The cases considered are $\theta_c = 0^o$ (perfect wetting) and 38^o. These and other that will be presented at the congress are discussed below.

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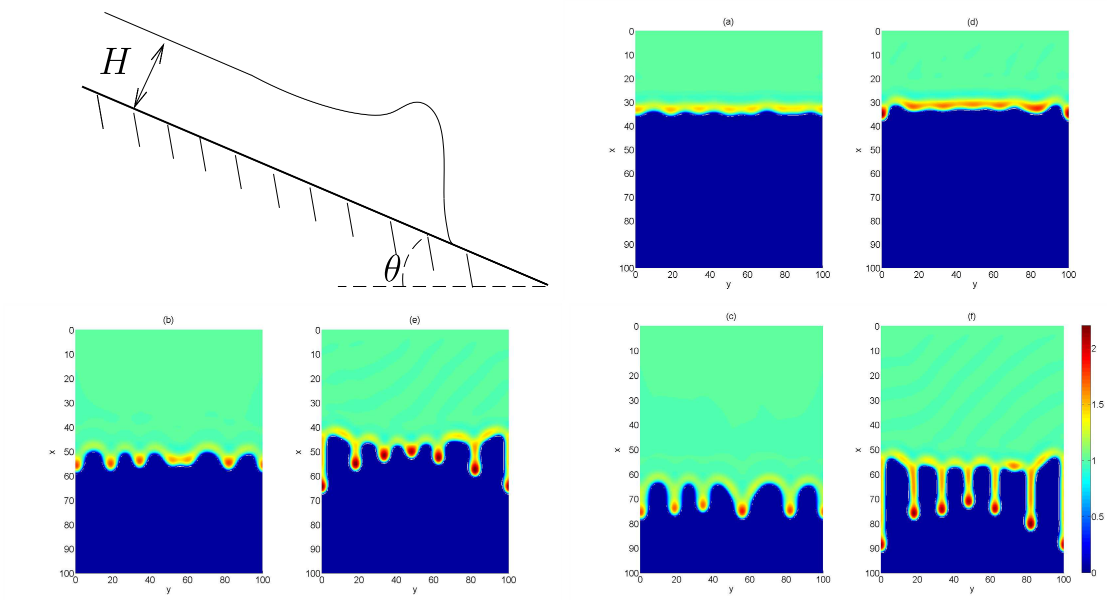

Figure 1. Schematic side view of a thin film (density, $\rho = 1075$ kg m^{-3}, surface tension, $\sigma = 0.069$ Pa m and viscosity, $\mu = 3.11 \times 10^{-2}$ Pa s) flowing down an inclined substrate, showing the advancing front (top left) and colour maps of the evolution of the front on a substrate, with $\theta = 85^o$, at $t = 25$ (top right), $t = 50$ (bottom left) and $t = 75$ (bottom right): (a)-(c) when the fluid is fully wetting; (d)-(f) when the fluid is partially wetting, $\theta_c = 38^o$.

In the case of perfectly wetting films, the predicted rivulet pattern at long times is found to be in very good agreement with observations made experimentally [7]; Huppert's [1] suggested linear fit with capillary length is recovered exactly. The shape, dynamics and growth of the rivulets are shown to be dependent on inclination angle. A relationship for the width of the emerging rivulets is found to correspond well to the findings of experiments. The merger of neighbouring rivulets is observed and is explored in detail (an example of two rivulets that are about to merge can be seen in Figure 1(b) which are then seen to have merged in Figure 1(c)); from the resulting long-time solutions generated it is possible to construct a map of rivulet wave length versus inclination angle (up to and including a vertically aligned substrate) which can be used to predict whether, as a particular film flow evolves with time, neighbouring rivulets will merge. The same is compared with a complementary linear stability analysis .

The results obtained for partially wetting fluids similarly lead to good agreement with experimental observation [7], while showing the differences between the dynamics of rivulet growth for partially and fully wetting fluids. Immediately observable disparities include the shape of the rivulets; for a partially wetting fluid the rivulets are straight edged and finger shaped as opposed to the saw-tooth pattern observed with highly wetting fluids. Rivulets elongate at a faster rate for a fluid with a high contact angle and are much thinner. Relationships for the width and wavelength of the rivulets are extracted from the solutions generated; that for the wavelength of the rivulet pattern suggests a dependence on a higher power of the capillary length than the one found for a fully wetting fluid.

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