STABILITY OF A CONDENSING LIQUID FILM OF A BINARY VAPOR MIXTURE

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Summary We model the dynamics of a condensing liquid film of a binary vapor mixture using the long-wave approximation and perform a linear stability analysis around a flat-film solution based on this model. There exists a certain critical thickness below which the basic state is stable and above which it is unstable. A significant difference in its values is found for a water-ethanol system if the temperature dependence of the mass transfer coefficient is considered, which was ignored in an earlier work.

INTRODUCTION

When a zeotropic binary vapor mixture is cooled on a substrate, it condenses and a liquid film emerges. If the surface tension of a one component of the mixture having a higher boiling point is stronger than that of the other component, the liquid film takes an inhomogeneous form such as a droplet one. This phenomenon was first observed by Mirkovich and Missen [1] and explained by Ford and Missen [2] as follows: a thinner part of the liquid film has a lower interface temperature. Since the concentration of the component having a lower boiling point becomes higher for lower temperatures from the phase equilibrium relation, the surface tension is weakened at a thinner portion. Thus, a stronger surface tension at a thicker part pulls the liquid away from a thinner one and the instability develops. Stability of a flat-film state of the liquid film in such a system was investigated by Hijikata et al. [3] However, they disregarded an effect of mass gain of the liquid which accompanies condensation.

PROBLEM FORMULATION

In order to analyze such a phenomenon, we apply the long-wave approximation to the condensing liquid film and derive a nonlinear partial differential equation describing the spatio-temporal evolution of the film thickness. Here we adopt an interfacial boundary condition which takes account of an effect of mass gain mentioned above. Furthermore, in Ref. [3] a phenomenological law that the condensation mass flux J is proportional to the difference between the vapor mass concentration at the interface c_{vI} and the ambient one c_0 was used:

$$J = \beta(c_{vI} - c_0), \tag{1}$$

where β is the mass transfer coefficient. Since c_{vI} is a function of temperature through the assumption of local thermodynamic equilibrium, the authors of Ref. [3] linearized Eq. (1) with respect to temperature as follows:

$$J = \beta \frac{dc_{vI}}{dT}(T_I - T_0), \tag{2}$$

where T_I and T_0 are the interface and ambient temperatures. Nevertheless, in their formulation (2) the temperature dependence of the mass transfer coefficient β itself was not considered. In fact, it is shown that β does depend on the temperature as follows. The condition of mass flux balance of a one component of the mixture at the liquid-vapor interface reads

$$c_l J = \rho_v D_v \nabla c_v \cdot \boldsymbol{n} + c_{vI} J, \tag{3}$$

where c_l denotes the liquid mass concentration, ρ_v the vapor density, D_v the mass diffusivity in the gas phase and $\nabla c_v \cdot \boldsymbol{n}$ the differentiation of the vapor mass concentration in the direction normal to the interface. In Eq. (3), we have neglected the gradient term of the liquid concentration. Equation (3) can be rewritten into the expression for J,

$$J = -\frac{\rho_v D_v \nabla c_v \cdot \boldsymbol{n}}{c_{vI} - c_l}. \tag{4}$$

Regarding the differentiation term in Eq. (4) we employ the approximation

$$-\nabla c_v \cdot \boldsymbol{n} \simeq \frac{c_{vI} - c_0}{\delta}, \tag{5}$$

where δ is a constant of dimension of length determined from the ambient-vapor condition. Using Eq. (5) it is found that the mass transfer coefficient β corresponds to $\rho_v D_v/(c_{vI} - c_l)\delta$ by comparison between Eqs. (1) and (4). Even if we regard the material properties ρ_v and D_v as constant, $c_{vI} - c_l$ varies with the interface temperature owing to the phase equilibrium relation. In our model, we incorporate the temperature derivative term of the mass transfer coefficient arising from the phase equilibrium relation at the liquid-vapor interface.

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RESULTS

Based on our model, we performed a linear stability analysis around the basic state where the flat liquid-vapor interface moves upward by condensation. We found that there exists a certain critical thickness below which the basic state is stable and above which it is unstable. The effect of mass gain included into our model stabilizes a disturbance and dominates over the destabilizing effect of solutocapillarity in the stable region. The critical thickness is determined from the balance between both the effects and inversely proportional to the temperature derivative of the condensation mass flux, which depends on both the equilibrium vapor concentration and the mass transfer coefficient from the above discussion. Therefore, the value of the critical thickness becomes different whether or not the effect of the temperature variation of the mass transfer coefficient is taken into account. We plot its values as a function of the wall temperature for a water-ethanol system in the left panel of Fig. 1. The significant difference in the values of the critical thickness can be observed for lower concentrations of ethanol if the temperature dependence of the mass transfer coefficient is considered.

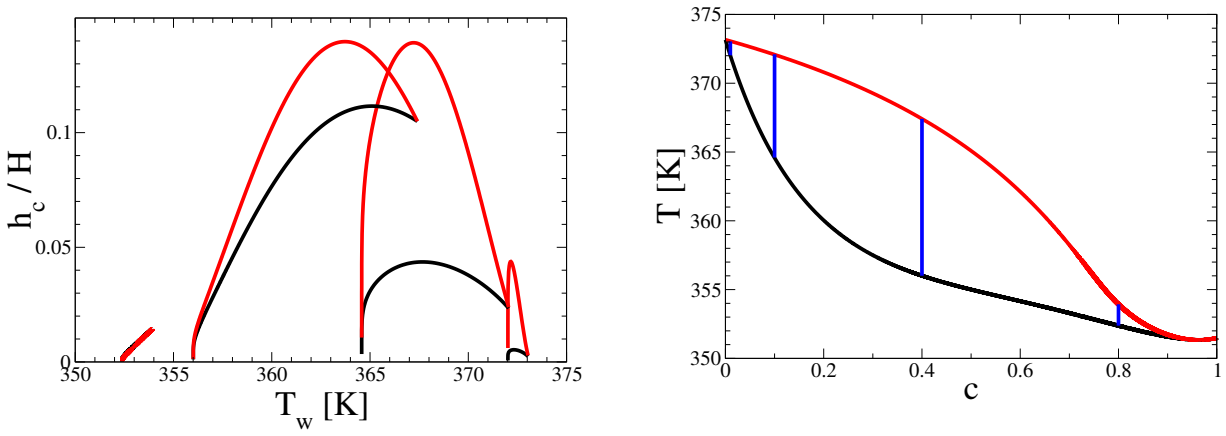

Figure 1. Left: The critical thickness h_c divided by H, which is the depth of the vapor layer where the concentration can vary, versus the wall temperature T_w for a water-ethanol mixture system at atmospheric pressure. The red curves correspond to the cases where the mass fractions of ethanol of the ambient-vapor mixture are 0.01, 0.1, 0.4 and 0.8 from left to right. The black curves represent the corresponding cases where the temperature dependence of the mass transfer coefficient is ignored as in Ref. [3]. Each pair of the curves is plotted in the temperature domains between the liquid and vapor lines, indicated by the blue lines in the right panel. Right: The vapor-liquid phase equilibrium diagram for water-ethanol mixture at atmospheric pressure. The black and red curves are the liquid and vapor lines. The composition c represents the mass fraction of ethanol.

Cutoff wavenumber

Since disturbances of short wave or large wavenumber always decay due to the effect of surface tension, we can define the cutoff wavenumber below which modes are amplified. This cutoff wavenumber is a function of the film thickness and increases until a certain thickness and decreases above it. Hence, the maximum cutoff wavenumber exists. For a water-ethanol system, we show that the wavenumber of patters of the liquid film observed in the experiment [3] has the same parameter dependence as that of the maximum cutoff wavenumber predicted from our model.

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