# SELF-SIMILAR THERMALLY DRIVEN RUPTURE OF THIN LIQUID SHEETS

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*Summary*

We investigate the application of Marangoni surface stresses in controlling the rupture of a symmetrically heated thin viscous liquid sheet. Varying the physical parameters associated with the system allows selection of the time and location of rupture (and even some control the number of simultaneous rupture points). We argue that Marangoni stresses can generate rupture of the sheet, even without the inclusion of van der Waals effects and we identify a first-kind similarity solution that governs the small-scale rupture process.

## PROBLEM DESCRIPTION

The rupture of sheets and threads and films, in a lubrication theory context (see, for example, [1]), has been investigated in a variety of scenarios, including the use of surfactants [2], electric fields (as considered in [3], [4], and [5]), nematic liquid crystalline jets (see [6]), and van der Waals forces ([7] and [8]). We consider the scenario illustrated in Figure 1, where a symmetric fluid sheet of average thickness $2\,h_o$ is moving at an average speed $U_{mean}$ from left to right. We define a coordinate system in relation to the centerline of the sheet, $z = 0$, as shown in Figure 1. At time $t = 0$, a prescribed temperature distribution $T = T_o + \Delta T\theta(x, 0)$ (where $T_o$ is the ambient temperature) is imposed on the fluid sheet.

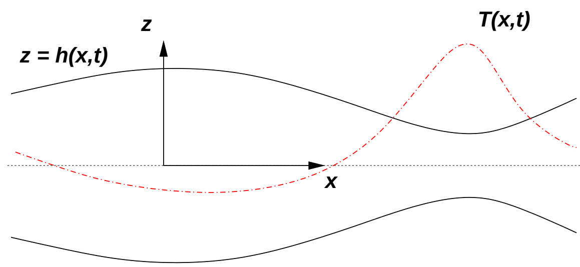


**Figure 1.** A sheet of incompressible fluid is excited at time $t = 0$ by an imposed temperature distribution $T(x, 0)$; note that we take the initial temperature distribution to be independent of the $z-$coordinate.

We investigate the case where fluid pressures are balanced by capillarity and viscous normal stresses, and the remaining effects of thermocapillarity, fluid inertia, van der Waals forces and thermal heat loss through the interface also balance. Employing a standard lubrication approximation, the (non-dimensionalised) governing equations for the dynamics of the one-dimensional heated symmetric fluid sheet (taking gravitational effects to be negligible) are consequently found to be

$$h_t + (uh)_x = 0\,, \tag{1}$$

$$Pr\,(\theta_t + u\theta_x) = \frac{1}{h}\,[h\theta_x]_x - \frac{B\theta}{h}\,, \tag{2}$$

$$u_t + uu_x = \frac{4}{h}\,[hu_x]_x + Sh_{xxx} - M\frac{\theta_x}{h} + \frac{3\,K}{8h^4}\,h_x, \tag{3}$$

where $u$ is the $x$-component of fluid velocity, $K$ is the scaled Hamaker constant, $Pr$ is the Prandtl number, $M$ is the Marangoni number, $S$ is a surface tension parameter (the ratio of capillary to viscous forces) and $B$ is the Biot number.

### Large-scale dynamics

We apply a long-wave analysis in the limit where deviations from the mean sheet velocity are small, but thermocapillary stresses and heat transfer from the sheet to the environment are significant. From a linear stability analysis, we find that a stable thermal mode couples the velocity and interfacial dynamics. We also find that the phase difference between the initial temperature and velocity distributions is a key parameter in determining the nonlinear stability of the fluid sheet and consequently the time of rupture. In particular, rupture can be induced thermally even in cases when the heat transfer to the surrounding environment is significant, provided that the initial phase shift between the initial velocity and temperature disturbances is close to $\pi/2$.

### Self-similar rupture

The work [8] identified a first-kind similarity solution controlling the rupture of an isothermal liquid film driven by van der Waals forces. In the absence of van der Waals forces, the similarity solution is no longer valid, corresponding to the absence of rupture in this case. When $K \neq 0$ (corresponding to the inclusion of van der Waals effects), the system (1)-(3)

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analogously admits a first-kind self-similar solution local to the rupture point in time and space. However, somewhat surprisingly, we will show that a first-kind self-similar solution can be found here even when van der Waals forces are neglected, namely

$$h \sim (t_c - t)\hat{h}(\eta), \quad u \sim (t_c - t)^{-1/2}\hat{u}(\eta), \quad \theta \sim \hat{\theta}(\eta), \quad \eta = \frac{x}{(t_c - t)^{1/2}};$$

see Figure 2 for illustrative solutions of (1)-(3) approaching rupture with $K = 0$. Furthermore, we note that the similarity solution continues to hold, even for relatively large Biot numbers and may therefore have relevance, for example, to industrial manufacturing processes (which often suffer substantial heat loss to the external environment).

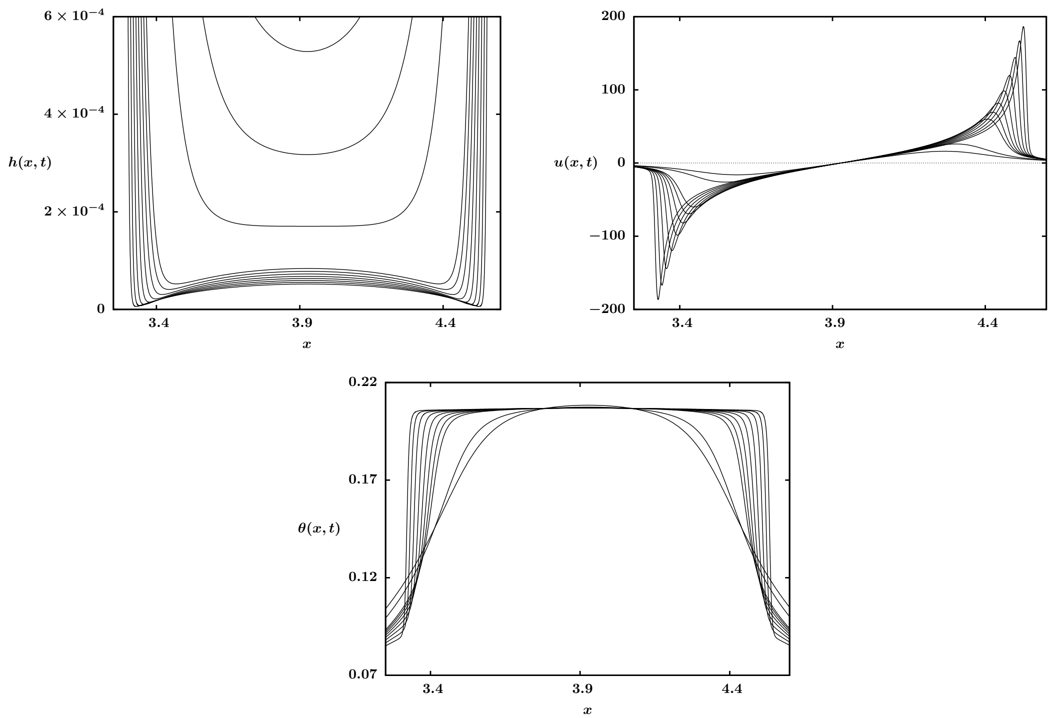


**Figure 2.** Solutions to (1)-(3) with $K = 0$ as $t$ approaches the finite rupture time $t_c$.

## References


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