THERMO-ELECTRO-HYDRODYNAMIC INSTABILITIES OF A DIELECTRIC LIQUID IN A CYLINDRICAL ANNULUS

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Summary : The critical conditions of the destabilization of a dielectric fluid confined in a vertical annulus between two infinitely long coaxial stationary cylinders under zero gravity conditions have been determined using linear stability analysis and direct numerical simulations. A radial temperature gradient and a high alternating electric field induce an effective gravity that can be used to create convection under microgravity conditions. The critical modes are non axisymmetric and made of stationary helices. Just above the onset, the modes exhibit a time dependent behavior.

Motivation and problem formulation

The dielectrophoretic force is a body force observed when a dielectric liquid is permeated by an inhomogeneous electric field. We consider the configuration of an annular gap filled by a dielectric liquid, with both a radial temperature gradient and a radial alternating electric field. In such case, temperature dependency of the permittivity induces an inhomogeneity of the electric field, and results in dielectrophoretic force [1, 2]. This force is analogous to Archimedian force with an effective gravity which is parallel to the gradient of electric energy. The dielectrophoretic force generates an thermo-electro-hydrodynamic convective instabilities. The resulting flows may enhance heat transfer. This is of particular interest for space applications under weightless condition where the natural convection cannot appear and forced convection from moving parts such as pumps is undesirable due to the expected wearing and lower reliability. Hence heat exchanger relying on TEHD instabilities could provide light, compact and reliable heat exchanger for aerospace cooling systems [3]. We present numerical results on the heat transfer in a cylindrical annulus submitted to a radial alternative electric field and a radial temperature gradient. The stability of this flow is numerically investigated. The flow is assumed to be incompressible and Newtonian, the timescales are chosen in such a way that the electric field can be replaced by an effective electric field assumed irrotational. The governing flow equations are then the Navier-Stokes equations coupled to energy equation and electric field equations [4] :

$$\vec{\nabla}\cdot\vec{v}=0 \quad ; \quad \rho\left(\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\vec{\nabla})\vec{v}\right)=-\vec{\nabla}p+\mu\nabla^2\vec{v}+\vec{f}_E$$

$$\frac{\partial T}{\partial t}+\vec{v}.\vec{\nabla}T=\kappa\,\Delta T \quad ; \quad \vec{\nabla}.\left(\varepsilon\vec{E}\right)=0 \quad ; \quad \vec{\nabla}\times\vec{E}=\vec{0}$$

where ρ, μ , κ and ε represent is the density, the dynamic viscosity, the thermal diffusivity and the dielectric permittivity of the liquid respectively, $\vec{v}$, p and T represent the velocity , the dynamic pressure (including an electric contribution) and the temperature of the flow respectively, $\vec{E}$ is the electric field with frequency large compared to frequencies associated to fluid timescales. The dielectrophoretic force is given by [1] : $\vec{f}_E=-\vec{E}^2\,\vec{\nabla}\varepsilon/2$.

We assume the Boussinesq approximation for the temperature dependence of the density and permittivity as follows :

$$\rho=\rho_0[1-\alpha(T-T_0)]\ ;\ \rho_0=\rho(T_0) \quad ; \quad \varepsilon=\varepsilon_1[1-\gamma(T-T_0)]) \quad ; \varepsilon_1=\varepsilon(T_0)$$

From the above equations, one deduces the vorticity equation in the form

$$\frac{\partial\vec{\omega}}{\partial t}+\left(\vec{v}.\vec{\nabla}\right)\vec{\omega}=\left(\vec{\omega}.\vec{\nabla}\right)\vec{v}+\nu\,\Delta\vec{\omega}+\alpha\left(\vec{g}+\vec{g}_e\right)\times\vec{\nabla}T \ ; \ \vec{\omega}=\vec{\nabla}\times\vec{v}$$

where $\vec{g}_e$ the *electric gravity*. In case of the base state in a cylindrical annulus submitted to the tension ΔV, the electric gravity reads

$$\vec{g}_e=\frac{\varepsilon_1\gamma}{\rho_0\alpha}\left[\frac{V_e}{\ln(a/b)}\right]^2\frac{\vec{e}_r}{r^3} \quad , \quad V_e=\frac{\Delta V}{\sqrt{2}}$$

The system is spanned by a parameter space defined by the aspect and radius ratio, the Prandtl number, the thermal and electric Grashof number. The numerical simulations of the flow equations were performed for the case of zero Earth gravity (g = 0) in order to isolate the electrical gravity effect [2, 4]. For fixed values of the aspect ratio and the Prandtl

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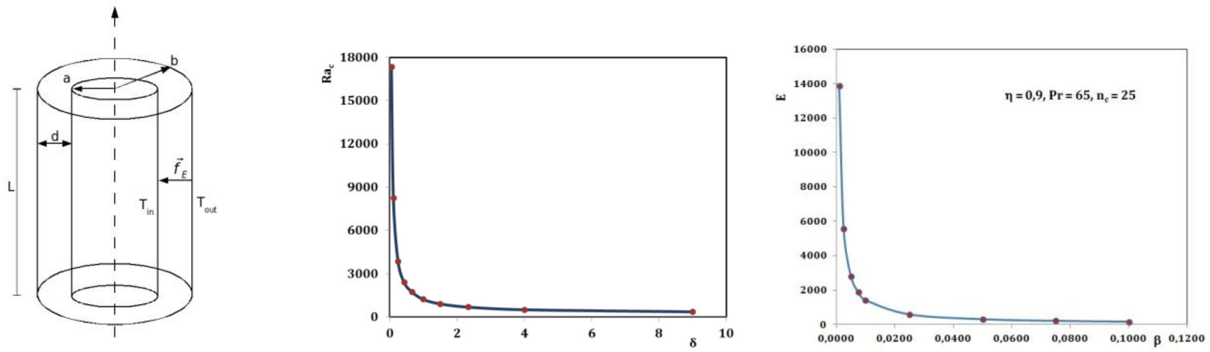
number, we have determined the critical electric field to be applied in order to initiate the convective motion in the annulus for different values of the temperature difference [5] in the weightless conditions, i.e. $\vec{g} = \vec{0}$.

RESULTS

In order to interprete the results obtained from numerical simulations and linear stability analysis, we defined the electric Rayleigh number $Ra = \alpha \delta T g_e d^3 / \nu\kappa$, the scaled electric tension $E = g_e d^3 / \nu^2$ together with the scaled temperature gradient $B = \alpha \delta T$. The main results can be summarized as follows :

1) The critical modes are stationary inclined modes with an axial and an azimuthal wavenumber n.
2) The values of the critical parameters (Rayleigh number, axial wavenumber and azimuthal wavenumber) are independent on the fluid nature i.e. on the Prandtl number as in the case of Rayleigh-Bénard problem, but they vary with the curvature $\delta = d/a$ of the annulus.
3) The critical value of the scaled effective tension imposed on the cylinders decreases as the radial temperature gradient increases.



a) b) c)

Figure : a) Scheme of the cylindrical annulus under dielectrophoretic effect; b) Dependence of the critical Rayleigh number with the curvature; c) Dependence of the critical value of scaled applied effective tension with the scaled temperature gradient.

CONCLUSION

We conducted the linear stability analysis of dielectric fluid confined between two coaxial cylinders subjected to temperature gradient and high alternating electric field. The critical parameters depend on the radius ratio η and are independent of the fluid considered. The critical modes are stationary helices with an inclination that is independent of the Prandtl number *Pr* and depend only on the radius ratio η. A further study is ongoing for the effect of the ground gravity and the rotation of the inner cylinder. A similar study has been performed in a spherical shell with a differential rotation in GEOFLOW experiment on International Space Station by the group from the Brandenburg University of Technology at Cottbus and the results are under analysis [6].

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