NONLINEAR PSE FOR THREE-DIMENSIONAL STRUCTURES IN SPATIALLY DEVELOPING MIXING LAYERS

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Summary The method of nonlinear parabolized stability equations (PSE) is applied in the simulation of vortex structures in supersonic mixing layer. Evolutions of the unstable two- and three-dimensional disturbances are investigated using spatial marching method. The instantaneous flow field is obtained by adding the harmonic waves to basic flow. The nonlinear stability and flow structure of H-type transition is analyzed and simulated. The results demonstrate that the nonlinear PSE methods can rapidly capture the three-dimensional large scale nonlinear structures at the early stage of transition.

INTRODUCTION

Mixing layer flow is a complicate, unsteady and nonlinear fluid dynamics issue. The study of the mixing mechanism has great theoretical and engineering importance, such as to the enhancement of mixing efficiency in scramjet, the suppression of the aero-optic phenomenon in guided missiles[1] and the radiation of aeroacoustic noise in airlines[2]. As the flow structure of mixing layers is dominated by the instability waves[3], the stability analysis of disturbance forms the basis of the active and passive control of mixing layer.
The method of parabolized stability equations (PSE) is very different from the direct numerical simulation (DNS) which resolves all scales of flow quantities or disturbance quantities. The PSE method decomposes the disturbances into multiple harmonic modes in Fourier space by fast Fourier transform[4]. The assumption of slow-varying in streamwise direction enables the stability equations to be solved by spatial marching method. This leads to a highly efficient simulation of flow fields. Because the approximations of locally-parallel flows and small disturbances are not employed, the PSE method can deal with the nonparallel and nonlinear effects, and hence is an effective way for the stability analysis of mixing layer.
The nonlinearity of governing equations of fluid mechanics is a primary source of many difficulties. It is the nonlinearity that leads to turbulent motion. In this paper, a nonlinear PSE (NPSE) method is adopted in the simulation of three dimensional nonlinear structures at the early stage of transition in supersonic mixing layer flows.

METHODOLOGY

The instantaneous quantities in the compressible Navier-Stokes equations are decomposed into basic quantities and fluctuating quantities, and then the fluctuating components are modeled with the general form of traveling waves. When the slow-varying property is introduced, we can obtain the following nonlinear parabolized stability equations[5]

$$\left[\hat{D}_{(m,n)}+\hat{A}_{(m,n)}\frac{\partial}{\partial x}+\hat{B}_{(m,n)}\frac{\partial}{\partial y}+V_{yy}^{l}\frac{\partial^2}{\partial y^2}\right]\psi_{(m,n)}=\frac{[F^n]_{(m,n)}}{A_{(m,n)}\chi_{(m,n)}};\quad m=(-\infty,+\infty)\quad n=(-\infty,+\infty)$$

Where subscript '(m,n)' denote the two dimensional Fourier modes in time and spanwise directions. $\psi_{(m,n)}$ are the shape functions, including the shape functions of velocity, temperature, density and pressure. Coefficient matrices, namely $\hat{D}_{(m,n)}, \hat{A}_{(m,n)}, \hat{B}_{(m,n)}$ and V_{yy}^{l}, are related to the basic quantities, mode frequencies and mode wavenumbers. The nonlinear terms F^n demonstrate the Fourier modes interacting each other. And hence governing equations of disturbances are transformed to a system of mode equations.
We keep two order terms in F^n, so that the nonlinear action on any mode (m,n) can be expressed as the sum of the interactions between any two modes (m_1,n_1) and (m_2,n_2) if $m_1+m_2=m$ and $n_1+n_2=n$ and can be written as following

$$[F^n]_{(m,n)}=\sum_{m_1=-\infty}^{+\infty}\sum_{n_1=-\infty}^{+\infty}N(m_1,n_1,m_2,n_2)\cdot A_{(m_1,n_1)}A_{(m_2,n_2)}\cdot\chi_{(m_1,n_1)}\chi_{(m_2,n_2)}$$, where $m_2=m-m_1$ and $n_2=n-n_1$

Grid stretching is used before the solving of PSE to offer high resolution of the flow in the shear region for finite mesh point number[6]. After that, forth order central differential method is used for discretization in normal direction, while first order backward differential formula is chosen for streamwise derivative.
The constraint is usually imposed on the streamwise growth of ψ to eliminate the ambiguity in streamwise direction. The present investigation adopt the following formula to constrain the streamwise change of ψ

$$\int_{-y_{\max}}^{y_{\max}}(\tilde{v})_m^{+}(\tilde{v}_x)_m\,dy=0$$

Basic laminar flow and initial disturbances should be given before the spatial marching procedure. The compressible similar boundary layer equations are used as the basic flow of mixing layer flow. The results of LST are used as the initial condition of main disturbances. For high order harmonic waves which is usually linear stable, a parallel version of

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nonlinear stability equations without streamwise derivatives and vertical velocity component are solved to get their initial conditions.

RESULTS

First we investigate the ability of nonlinear PSE in capturing the three dimensional structure in the early stage of transition such as Λ vortex. The basic flow condition are as follows, Reynolds number is 5000, incoming Mach number M_1=2.5 and M_2=1.5 respectively, and the temperature ratio is 1.0. At the inlet the unstable disturbance with frequency ω=0.8 and its three-dimensional subharmonic disturbance with spanwise wave number β=0.5 are imposed. As shown in Fig.1, the H-type Λ vortex structures in the early stage of transition obtained by the NPSE method are qualitatively consistent with that from DNS calculations[7]. It is suggested that the PSE can well capture the Λ vortex structure.

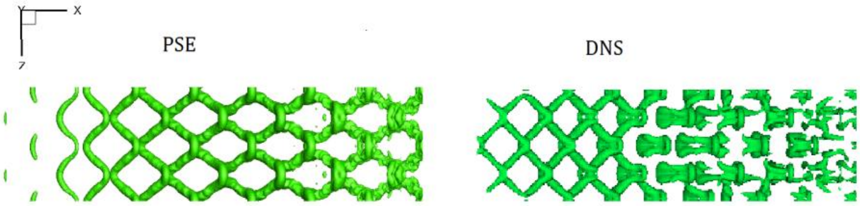

Figure 1 Comparison of Λ structure form nonlinear PSE and DNS (vertical view)

It is important to investigate the nonlinear development of disturbances and their roles in the formation of vortex structure. In the next case, we consider the same basic flow as the previous one. The most unstable two-dimensional disturbance with frequency ω=0.5781 is imposed at the inlet, and the amplitude is set to 0.005. The three-dimensional subharmonic disturbance with spanwise wave number β=0.55 is imposed to excite the three dimensional structure, and the amplitude is set to 0.001. The evolution of mode energy is shown in Fig.2. At the inlet, the two-dimensional fundamental mode is most unstable. It grows rapidly to a high level near the region x=50, which leads to the arisen of two-dimensional vortex tubes in mixing layer, as the low pressure iso-surface[8] shown in Fig.3. In the downstream, the two dimensional disturbance saturates due to the nonlinear interaction, the amplitude decreases slowly, and the energy transfers from low average flow and low frequencies modes to high order modes. At the following regions, the three-dimensional subharmonic mode grows to the high level, and the vortex tubes starts to distort and Λ structures emerge which distribute periodically in the spanwise direction and stagger in the streamwise direction. It is a typical phenomenon of H-type transition. The mode (0,0) with zero frequency and is produced by the nonlinear interaction of modes, increases the thickness of mixing layer and enhances the mixing efficiency. High order harmonic modes which are linear stable get into nonlinear stage initially and grow rapidly.

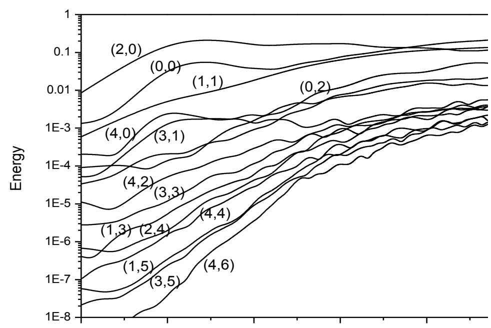

Figure 2 Mode energy evolution.

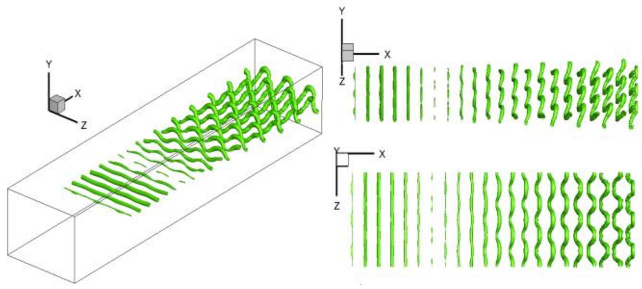

Figure 3 Low pressure iso-surface.

CONCLUSION

This research shows that NPSE method is a rapid and effective method for the simulation of the nonlinear evolution of disturbances and large scale vortex structures in supersonic plane mixing layers.

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