THREE-DIMENSIONAL FINITE-AMPLITUDE SOLUTIONS IN ROTATING POISEUILLE FLOW

D. P. Wall[a)] & M. Nagata
Dept of Aeronautics & Astronautics, Graduate School of Engineering, Kyoto University, Yoshida-Honmachi, Sakyo-ku, Kyoto. 606-8501 Japan.

Summary Exact three-dimensional finite-amplitude solutions are presented for the problem of flow through a parallel-sided channel subject to spanwise rotation. A number of distinct flows are identified, with streamwise flow structures featuring low-speed streaks in streamwise velocity, which may be varicose or sinusoidal, and which are flanked by aligned or staggered streamwise vortices. Results for plane Poiseuille flow, the special case of the present problem in the limit of vanishing rotation, are also presented.

BACKGROUND

The effect of rotation on fluid dynamics is of prime importance to understanding fluid behaviour in geophysical problems, oceanography and meteorology, and also features in many engineering problems such as flow in rotating machinery.
The present problem considers channel flow driven in the streamwise direction by a constant imposed pressure gradient and subject to system rotation about a spanwise axis. Experiment and simulation studies find that with increasing Reynolds number (or rotation rate for a large enough Reynolds number) the basic flow first loses stability to a steady streamwise-independent streamwise vortex secondary flow, which itself then loses stability to travelling wave tertiary flows, which feature a waviness imposed on the vortical structures of the secondary flow. Finite-amplitude solutions describing the two-dimensional secondary flows have previously been presented by Wall & Nagata [3]. In the present study we seek exact finite-amplitude solutions for the three-dimensional travelling wave flows that bifurcate from these secondary flows, and for completeness also seek three-dimensional travelling wave secondary flows that bifurcate from the basic state.

FORMULATION

Flow through the channel is governed by the Navier-Stokes equations for an incompressible fluid expressed in a rotating frame. All lengths are scaled by half the channel width L, and the velocity by ν/L, where ν denotes the kinematic viscosity. Two physical parameters appear in the problem: the Reynolds number, $R = L^3J/2\rho\nu^2$, and rotation number, $\Omega = 2\Omega^* L^2/\nu$, where $-J$ ($J > 0$) is the pressure gradient in the positive streamwise direction, ρ is the constant fluid density and Ω^* denotes the dimensional rotation rate. We seek nonlinear solutions to the governing equations that deviate from the basic state. Solution variables are expressed as Fourier series in the spanwise (y) and streamwise (x) directions, and in terms of modified Chebychev functions in the wall normal direction (z). The amplitude coefficients are then obtained by Newton iteration. Henceforth (L, M, N) denote the number of modes retained in the (z,x,y) directions.

RESULTS

Three-dimensional Secondary Flows

Experiments find that the basic flow first loses stability to two-dimensional streamwise-independent secondary flows, as studied in [3]. We also, however, consider three-dimensional secondary flows bifurcating directly from the basic state since such flows have also been observed in simulation studies [4], and may play an important role in turbulent flow regimes. Only a single symmetry group is possible for these three-dimensional secondary flows, with the solutions satisfying a reflectional symmetry about a streamwise - wall normal plane, and a glide-reflectional symmetry about streamwise-wall normal planes located a quarter of one spanwise wavelength either side of the plane of reflectional symmetry. The streamwise velocity components of velocity and vorticity for a representative flow of this type are shown in Fig. 1. The flow is distinguished by two lines of low-speed pulses per spanwise wavelength, which are flanked by aligned vortices. The flow appears to be qualitatively similar to ribbon-cell finite-amplitude flows observed in Taylor-Couette flow.

Three-dimensional Tertiary Flows

Three-dimensional flows can also be found from bifurcations from the two-dimensional streamwise-independent secondary flows described in [3]. We consider spanwise superharmonic (i.e. the spanwise secondary disturbance Floquet parameter $b = 0$) and spanwise subharmonic ($b = \beta_{2D}/2$, where β_{2D} is the spanwise wavenumber of the secondary flow) secondary disturbances. It can be shown that two symmetry groups are possible for bifurcating superharmonic solutions. The velocity fields for flows of the first symmetry group, $\mathcal{A}$, satisfy a glide-reflectional symmetry about a streamwise-wall normal plane, while those of the second symmetry group, $\mathcal{B}$, satisfy a reflectional symmetry about the same plane. Only one symmetry group, $\mathcal{C}$, is possible for the subharmonic solutions, and this is found to be identical to that of the three-dimensional secondary flows described above. When the streamwise secondary disturbance d is $O(1)$ a single complex eigenmode loses stability on the upper branch of the secondary flow, before regaining stability on the lower branch of this flow. Fig. 1 shows such a superharmonic flow, of symmetry group $\mathcal{A}$, bifurcating from the upper branch of the secondary

[a)] Corresponding author. E-mail: d.wall@fx8.ecs.kyoto-u.ac.jp.

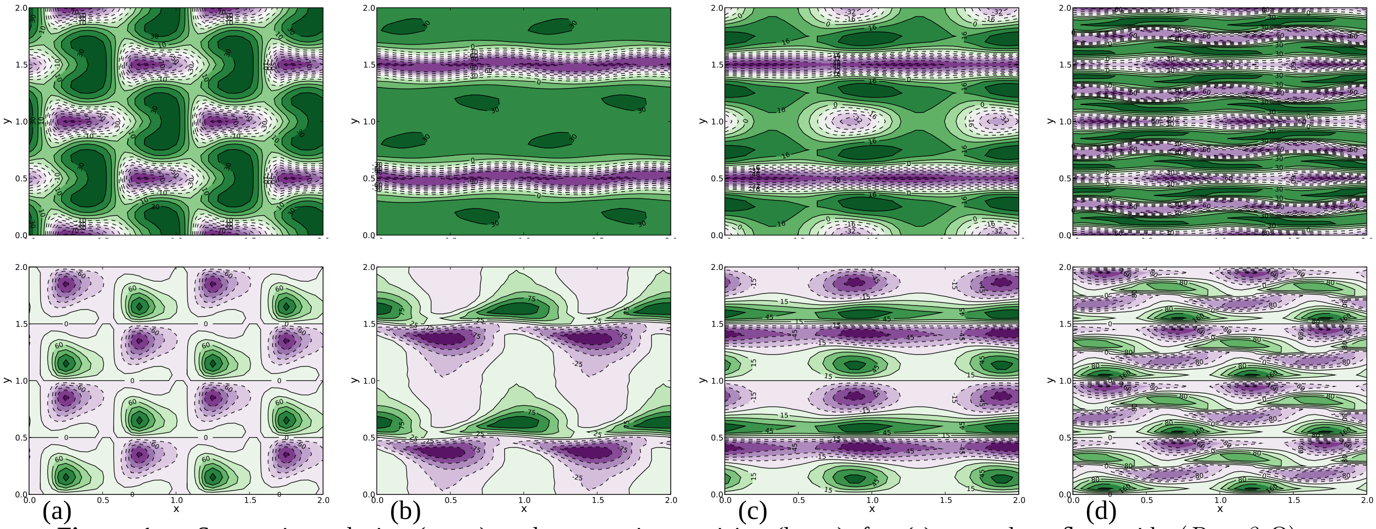

Figure 1. Streamwise velocity (upper) and streamwise vorticity (lower) for (a) secondary flow with $(R, \alpha, \beta, \Omega) = (315.7, 0.4, 2.89, 22.1325)$, (b) upper-branch superharmonic tertiary flow for $(R, \alpha, \beta, \Omega) = (430, 1.5, 2.5, 22.1325)$, (c) lower-branch superharmonic tertiary flow for $(R, \alpha, \beta, \Omega) = (267.8, 0.4, 2.5, 22.1325)$, (d) lower-branch subharmonic tertiary flow for $(R, \alpha, \beta, \Omega) = (429.3, 1.0, 1.25, 22.1325)$. Solutions computed at truncation level (a) and (d) $(L, M, N) = (29, 8, 16)$, (b) and (c) $(L, M, N) = (29, 8, 8)$. (Here α and β denote wavenumbers in the x and y directions.)

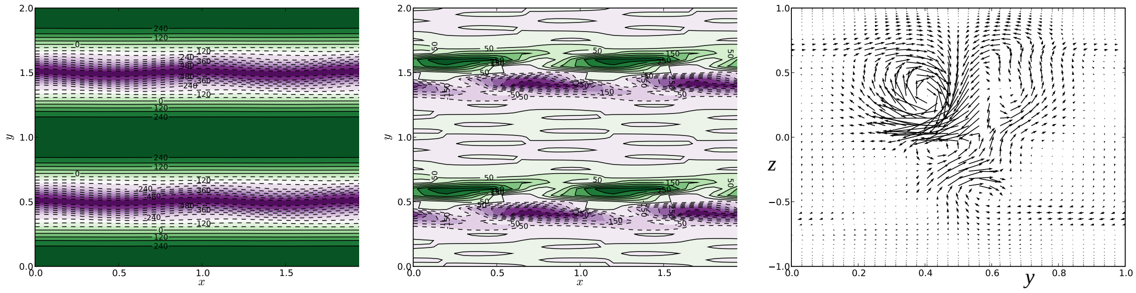

Figure 2. Solution for plane Poiseuille flow extended by homotopy from the flow shown in Fig. 1(b) for parameters $(R, \alpha, \beta, \Omega) = (2730, 2.896, 2.5, 0)$ at truncation level $(L, M, N) = (51, 9, 9)$. (left) Streamwise velocity, (middle) streamwise vorticity and (right) velocity vectors.

flow when d is $O(1)$. The flow is characterised by one low-speed sinusoidal streak in the streamwise velocity component u per spanwise wavelength, with the streaks flanked either side by staggered vortices. In contrast, superharmonic flows of the same symmetry group bifurcating from the lower branch of the secondary flow (not shown) feature two such low-speed streaks per spanwise wavelength.

When $d = O(10^{-1})$ up to four distinct eigenmodes can become unstable. The superharmonic flow shown in Fig. 1(c), arising when the second least stable eigenmode loses stability for $d = O(10^{-1})$, selects instead symmetry group $\mathcal{B}$. While similar to Fig. 1 (b) in that there is again one low-speed streamwise-orientated streak in u per spanwise wavelength, the streak is varicose rather than sinusoidal in form, and is accompanied by aligned rather than staggered vortices. A subharmonic flow bifurcating from the lower branch of the secondary flow when $d = O(1)$ is shown in Fig. 1(d), this flow exhibits four low-speed streaks per spanwise wavelength, two are sinusoidal flanked by staggered vortices and two are varicose flanked by aligned vortices.

Plane Poiseuille Flow

We are presently attempting to investigate whether the solution types found for the rotating Poiseuille flow problem described above can exist in the special case of zero rotation rate, i.e. in plane Poiseuille flow. Initial results find that the superharmonic solution type described in Fig. 1, can be transformed by homotopy to yield a new solution in plane Poiseuille flow. The solution is visualised in Fig. 2, and features two weaker vortices in the half of the channel ($-1 < z < 0$) that is stable according to Rayleigh's inviscid criterion in addition to the two stronger vortices in $0 < z < 1$. In contrast to Nagata & Deguchi's [1] and Waleffe's [2] solutions, the flow is not symmetric about the channel centreline.

References

[1] M. Nagata and K. Deguchi. Traveling wave solution with a $\wedge$-vortex in plane Poiseuille flow. Bifurcation and Instabilities in Fluid Dynamics Symposium, July 2011.

[2] F. Waleffe. Homotopy of exact coherent structures in plane shear flows. *Phys. Fluids*, 15:1517–1534, 2003.

[3] D. P. Wall and M. Nagata. Nonlinear secondary flow through a rotating channel. *J. Fluid Mech.*, 564:25–55, 2006.

[4] K. S. Yang and J. Kim. Numerical investigation of instability and transition in rotating plane Poiseuille flow. *Phys. Fluids*, A 3:633–641, 1991.