Exact solutions at the edge of turbulence

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Summary We present a combined experimental and numerical study of transitional pipe flow. The focus is on exact coherent structures in decaying turbulence which in the experiment is realized by an injection relaminarisation mechanism. The spatio temporal dynamics observed closely resemble those known from the edge state which forms the separatrix between turbulence and the laminar flow. Apart from matching streak vortex dynamics, the experimental structures also have the same axial advection speed as the edge state. Next the experimental velocity fields are used as initial conditions in direct numerical simulations. Applying a Newton method the experimental velocity fields are found to converge to unstable travelling wave solutions which are known to be embedded inside the edge.
In addition we report the computation of an exact unstable solution which is streamwise localised and shares key spatial features with turbulent puffs.

Background

Following the discoveries of unstable solutions in pipe and related shear flows it has been proposed that such unstable states provide the building blocks of turbulence[1,2]. In analogy to lower dimensional systems such unstable solutions are believed to form a chaotic attractor (or repeller) through secondary bifurcations and entanglement, giving rise to the turbulent dynamics. The laminar state on the other hand is known to remain stable for all (or at least up to very large) Reynolds numbers. Hence both states, laminar and turbulent, co-exist in the same Reynolds number regime. Which of the two states of fluid motion persists depends on initial conditions and perturbations of finite amplitude. Recent numerical studies showed that a single dynamical state the so called edge state[3] exists which separates laminar from turbulent motion.

Experimental and computational methods

Experiments are carried out in a pipe of D=30mm in diameter and 12 m in length. Velocity fields are simultaneously measured in two cross sections (perpendicular to the pipe axis) separated by 6D using two high speed stereoscopic particle image velocimetry systems. In each measurement plane velocity vectors (three components) are obtained in about 2500 points equally spaced over the cross section. Three dimensional velocity fields can be obtained by applying Taylor's hypothesis and converting the temporal information into spatial structures. In doing so we obtain full velocity fields which can be used as initial conditions for direct numerical simulations. The numerical code[4] is based on a solenoidal Petrov-Galerkin spectral scheme and we used a resolution of 33 x 77 x 27 axial x azimuthal x radial. A Newton-Raphson method was used to converge experimental flow states to exact travelling wave solutions.

Results

Using a method described in an earlier study [5] turbulent flow structures are relaminarised in the pipe experiment at Re=1900. Velocity fields are then measured at two streamwise locations in the decaying turbulence. The two measurement locations allow to determine the advection speed of the observed structures. The velocity fields found consist of two high speed streaks in the near wall region and a low velocity streak in the centre. Streamwise vortices are observed (see Figure 1 left panel) between the high and low speed regions. The vortices alternately become weaker and stronger in a quasi periodic fashion. Like for the numerically observed edge state[3], also the velocity field in the experiment is found to be asymmetric and the advection speed of the experimental flow structure is found to be identical to that of the edge state. Furthermore we were able to converge experimental three dimensional velocity fields to unstable travelling waves (an example of convergence to an asymmetric travelling wave[6] is shown in the right panel of Figure 1).

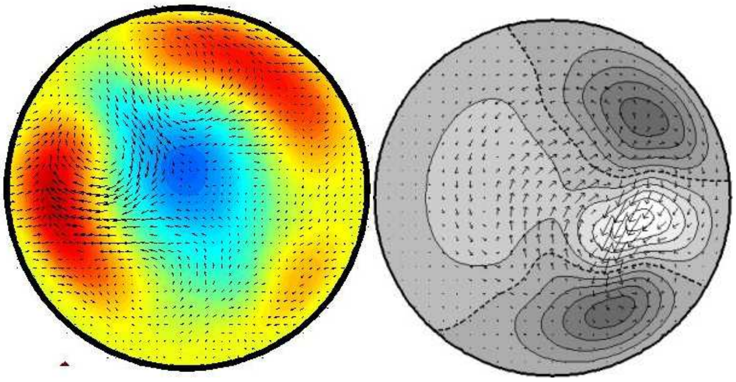

Figure 1: The left panel shows an averaged velocity field from the experiment, closely resembling the edge state in pipe flow. On the right hand side a cross section of an asymmetric travelling wave solution is shown.

CONCLUSIONS

We present the first experimental observation of the edge state in pipe flow experiments. In addition we demonstrate that experimental velocity fields in decaying turbulence can be successfully converged to exact solutions of the Navier-Stokes equations. This observation strongly supports the relevance of unstable solutions to the turbulent state and more specifically the laminar turbulent boundary.

References

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