WEAKLY NONLINEAR MODULATED TOLLMIEN-SCHLICHTING WAVES

Marcello A. F. Medeiros
Department of Aeronautical Engineering, University of São Paulo, São Carlos, SP, 13563-120, Brazil

Summary
The early nonlinear regime of modulated Tollmien-Schlichting waves was studied. An original set of artificially excited waves in wind tunnel was produce. In was found that these waves evolve through 3 different weakly nonlinear stages, namely, nonlinear direct mode-mode interaction, K-type instability and subharmonic-type instability. The packet phase relative to the wave envelope was found to affect only the third stage. Weakly nonlinear theory was applied for each stage and explained further aspects of the phase effect.

EXTENDED ABSTRACT

Natural transition involves modulated waves. Most of the knowledge about these waves come from studies of highly modulated pulse-excited wavepackets. Random excitations are in general less modulated, but these have not been systematically investigated. This paper is initially intended to contribute wind tunnel data on the topic. Figure 1 shows the evolution of waves with different levels of modulation emanating from an otherwise harmonic point source. The spanwise modulation was identical for all the signals. Clearly there is a pattern linking the nonlinear mean flow distortion of the top (unmodulated) signal to the nonlinearity in the pulse-excited wavepacket (the loss of a ripple), at the bottom. The flow studied developed on a flat plate and a loudspeaker excitation was introduced at 203 mm from its leading edge. The free-stream velocity was 17.3m/s. Measurements of the streamwise velocity only were carried out with a constant-temperature hot-wire anemometer at $y = 0.6\delta^*$ (close to the inner peak of a linear TS wave). Ensemble averaging provided a standard deviation bellow 2% of the velocity fluctuation. The experiment reproduce most conditions of a previous experiment on an unmodulated wave emanating from a point source [5], which provides further details. The experiments covered the whole disturbance in the spanwise direction. Frequency$\times$spanwise wavenumber spectra were calculated, figure 2.

The wave amplitudes were so small that transient growth effects were unimportant [2, 1, 6, 7, 8]. Previous works indicate that subharmonic resonance plays an important role in the nonlinear regime of pulse-excited wavepackets [2], and that this regime is very sensitive to the phase of the waves relative to the modulation envelope [6]. It is also shown that, besides the subharmonic instability, some mechanism of production of subharmonic waves is present [7]. Simultaneously with the subharmonic instability there are other nonlinear activities which also cannot be attributed to subharmonic resonance and have not yet been explained. These involves the the growth of waves with fundamental frequency and very high spanwise wavenumbers [1] and very low frequency signals at several spanwise wavenumbers [8]. It was here found that these activities were triggered by the spanwise modulation only. The first nonlinear stage was associated with directly forcing via mode-mode nonlinear interaction and produced longitudinal streaks. The spanwise wavenumber predicted by a weakly nonlinear model are shown by arrows for x=800mm in figure 2. The second nonlinear stage was K-type instability, which produced highly oblique fundamental modes and new streaks. The spanwise wavenumbers predicted by the secondary instability of the dominant two-dimensional wave is shown by arrows for x=1100mm in figure 2. For both stages the agreement is good. Similar spectral peaks were found for the streamwise modulated signals, albeit progressively noisier. These findings constituted the second contribution of the paper. The third nonlinear regime was subharmonic resonance. It was restricted to streamwise modulated signals, but even the least modulated signal displayed significant subharmonic content. The spanwise wavenumbers predicted by the triad resonance model [3] for the dominant two-dimensional wave are given by the $\times$ for x=1300mm (bottom signal).

Several phases relative to the envelope were tested. It was found that for all streamwise modulated signals the phase had strong effect, but it was restricted to the subharmonic nonlinear band, figure 2. A weakly nonlinear model developed by [4] explains that, owing to the modulation, a direct mode-mode interaction of the primary modes in the wavepacket produces the seeds for the subharmonic instability. Clearly this interaction is linked to the first nonlinear stage here described. The model yields strong phase dependence [4]. However, it is shown [6] that the most dangerous phase varies with the wavepacket amplitude, a feature that Craik's model could not explain. Here the model was extended by allowing the production of subharmonic seeds by the direct nonlinear interaction of the K-type modes too. Equations

$$\frac{d}{dt}d_+ = \lambda a_0 d_-^* + \mu_{A21} a_2 a_1^* + \mu_{C21} c_2 c_1^* \qquad \frac{d}{dt}d_- = \lambda a_0 d_+^* + \mu_{A21} a_2 a_1^* + \mu_{C21} c_2 c_1^*,$$

govern the evolution of a linearly neutral subharmonic wave pair (amplitude d). This is subject to the forcing provided by the nonlinear terms, a representing amplitudes of primary modes and c, K-type modes. a_0 is the amplitude of the dominant two-dimensional wave. In the packet several similar nonlinear forcing could produce these seeds. The equations present only one example, but it is enough to support the arguments raised here. For small amplitudes, the equations fall on the classical triad resonance [3]. If only the K-type modes are neglected, Craik's model [4] is recovered. Considering the way in which the waves were excited, the solution is

$$\frac{|d_\pm|}{e^{|\lambda a_0|t}} \approx \frac{|\mu_{A21} a_2 a_1|}{|\lambda a_0|}\left|\cos\left[\frac{1}{2}\arg(\lambda) + \frac{\phi_A}{2} - \arg(\mu_{A21})\right]\right| + \frac{|\mu_{C21} c_2 c_1|}{|\lambda a_0|}\left|\cos\left[\frac{1}{2}\arg(\lambda) + \frac{\phi_A}{2} - \frac{\Delta\phi}{2} - \arg(\mu_{C21})\right]\right|.$$

[a)] Corresponding author. E-mail: marcello@sc.usp.br

ϕ is the wave phase and $\Delta\phi$ is the phase difference between the fundamental and the K-type modes, and becomes constant under K-type resonance. If the first term dominates, the maximum amplitude of d occurs for $\phi_A = 2\arg(\mu_{A54}) - \arg(\lambda)$, and the minimum for $\phi_A = 2\arg(\mu_{A54}) - \arg(\lambda) \pm \pi$. If the second term dominates, maximum amplitude occurs for $\phi_A = 2\arg(\mu_{C54}) - \frac{\Delta\phi}{2} - \arg(\lambda)$. Since K-type modes are already nonlinear, the relative amplitude of the terms change as the amplitude of the primary packet changes, in qualitative agreement with the discussed phase effect.

Acknowledgement. Support to this work was provided by AFOSR/USA and CNPq/Brazil

References

[1] J. Cohen. *Phys. Fluids*, 6(3):1133–1143, March 1994.
[2] J. Cohen, K. S. Breuer, and J. H. Haritonidis. *J. Fluid Mech.*, 225:575–606, 1991.
[3] A. D. D. Craik. *J. Fluid Mech.*, 50:393–413, 1971.
[4] A. D. D. Craik. *J. Fluid Mech.*, 432:409–418, 2001.
[5] M. A. F. Medeiros. *J. Fluid Mech.*, 508:287–317, 2004.
[6] M. A. F. Medeiros and M. Gaster. *J. Fluid Mech.*, 397:259–283, 1999.
[7] M. A. F. Medeiros and M. Gaster. *J. Fluid Mech.*, 399:301–318, 1999.
[8] K. S. Yeo, X. Zhao, Z. Y. Wang, and K. C. Ng. *J. Fluid Mech.*, 652:333–372, 2010.

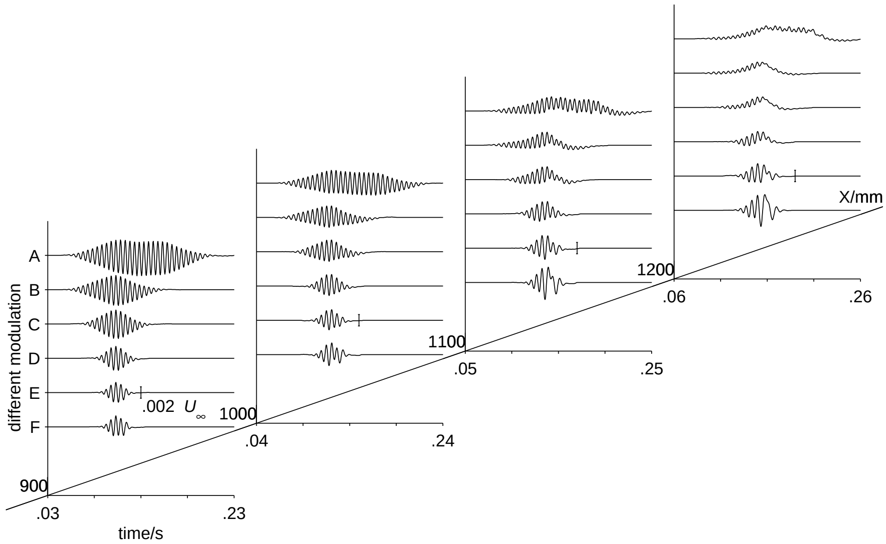

Figure 1. Evolution of TS waves with different modulation envelopes emanating from point source. Measurements along the centerline at $y = 0.6\delta^*$. An amplitude scale is given with signal E and remains constant for all stations.

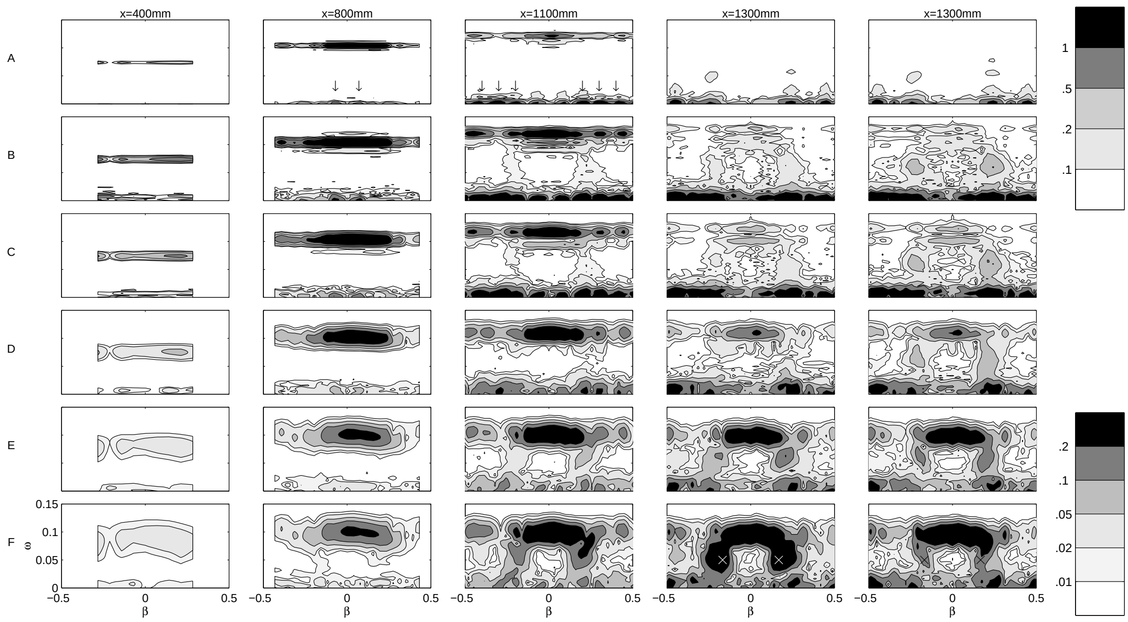

Figure 2. Frequency×spanwise spectra relative magnitude. The higher level scale is for signal A, the lower for the other signals. Arrows for signal A at x=800mm and 1100mm, and "×" for signals F at x=1300mm indicate nonlinear theoretical predictions. The wavepacket phase relative to the envelope is 0 except the right most frame for x=1300mm, which are for phase π.