BIFURCATIONS FOR AN INCOMPRESSIBLE FLOW PASSING OVER AN OPEN-CAVITY

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Summary
Several aerodynamics components involve open cavities as a fundamental part of design, manufacture and performances. Generic features of such configurations are characterized by flow induced oscillations which can excite the vibrational modes of the structure, leading to structural damage and a loss of aerodynamics performances. In an incompressible regime, two-dimensional shear layer oscillation above the cavity, the so-called shear layer mode, is typically described as an hydrodynamic feedback loop. Vortical structures generated at the leading edge of the cavity are amplified during their advection along the shear layer due to the Kelvin-Helmholtz instability followed by their impingement on the trailing edge. Nevertheless, several studies demonstrate a significant interaction between the external flow and the recirculating flow inside the cavity governed by three-dimensional large structure oscillating at low frequencies. The purpose of this work is to further explore the three-dimensional bifurcations of an incompressible open-cavity flow at low Reynolds number, below the threshold of the shear-layer mode, by means of DNS and a weakly nonlinear global stability analysis.

FLOW CONFIGURATION AND BIFURCATIONS

The cavity configuration and flow conditions are controlled by three parameters as depicted in Figure 1: the cavity aspect ratio L/D, the spanwise extend L_z/D and the ratio of the cavity lenght to the initial boundary layer displacement thickness at the leading edge of the cavity δ^*/D. A preliminary study focuses on cavity flow with $L/D = 1$, $L_z/D = 1$ and $\delta^*/D = 0.04$. The Reynolds number is based on U_∞ and L. A multidomains version of the DNS code presented in Chicheportiche etal [2] is used, where interfaces are solved through an influence matrix technique. The onset of spanwise bifurcation from a two-dimensional equilibrium state is illusrated in Figure 2 a) b). The latter is emphasized by the occurence of Taylor-Gœrtler vortices. A global linear stability analysis, based on a time-stepping technique, is performed. As shown in Figure 3, the most unstable mode is stationnary with a temporal amplification rate close to the one extracted from the DNS. A Fourier analysis along the spanwise direction indicates that the spanwise wave number $\beta = 2$ is selected for this spanwise extend. By increasing the Reynolds number, the flow is then destabilized by an unstationnary mode, characterized by a slow motion of vortices (see Figure 2 c)).

SUB- OR SUPERCRITICAL BIFURCATION: WEAKLY NONLINEAR STABILITY THEORY

The first bifurcation of a lid-driven cavity flow appears either sub- or supercritical, depending on the spanwise extend [3]. Such dynamics may occur in our case. For that purpose, let-us consider $Re_L^{-1} = Re_{L,c}^{-1} - \varepsilon$ with $0 < \varepsilon \ll 1$ and $Re_{L,c}$ the critical Reynolds number. Following Sipp & Lebedev [1], a multiple time scale analysis is considered, based on a fast time scale t and a slow time scale $t_1 = \varepsilon t$. The flow is thus decomposed as:

$$\mathbf{U}(t) = \mathbf{U}_0 + \sqrt{\varepsilon}\mathbf{U}_1(t, t_1) + \varepsilon\mathbf{U}_2(t, t_1) + \varepsilon\sqrt{\varepsilon}\mathbf{U}_3(t, t_1) + ...$$

Introducing such decomposition onto the Navier-Stokes equations yields equations at different order in ε. A Stuart-Landau amplitude equation is given by the $\varepsilon\sqrt{\varepsilon}$ order using a Fredholm alternative to avoid secular term:

$$\frac{dA}{dt} = \varepsilon\lambda A - \varepsilon\mu A|A|^2$$

where A denotes the amplitude of the leading global mode. The nature of the bifurcation could thus be analyzed theoretically by computing μ.

CONCLUSIONS AND OUTLOOKS

By assuming periodicity along the spanwise direction, DNS of a 3D incompressible flow passing over a square open cavity show that as the Reynolds number Re_L is increased, the flow undergoes a sequence of successive bifurcations inside the cavity. Each state is characterized by space and time symmetry breaking and the emergence of more complicated shapes. In particular, with a spanwise direction which extends one cavity lenght, the flow is first destabilized by a stationnary mode for a spanwise wavenumber $\beta = 2$. Hence, the bifurcation diagram will be investigate by means of DNS and a weakly nonlinear global stability analysis where several spanwise extends will be considered.

References

[1] Sipp, D. and Lebedev, A.: Global stability of base and mean flows: a general approach and its apllications to cylinder and open cavity flows. *J. Fluid Mech* **593**:333–358, 2007.
[2] Albensoeder, S. and Kuhlmann, H.: Nonlinear three-dimensional flow in the lid-driven square cavity *J. Fluid Mech* **569**:465–480, 2006.
[3] Chicheportiche, J., Merle, X., Gloerfelt, X. and Robinet, J.-C.: Direct numerical simulation and global stability analysis of three-dimensional instabilities in a lid-driven cavity *C.R Mecanique* **336**:586–591, 2008.

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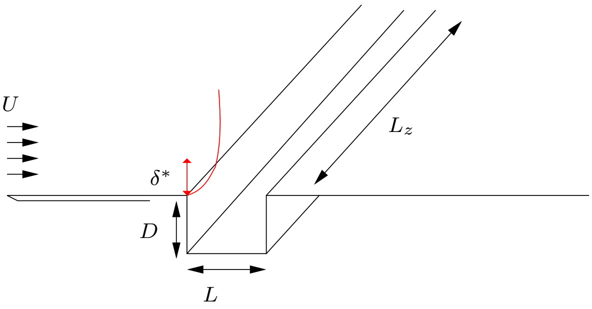
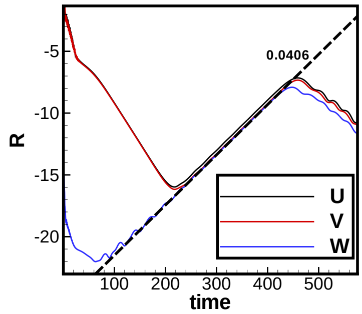
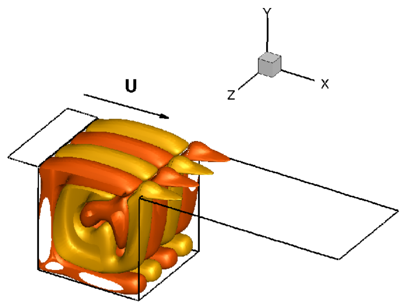


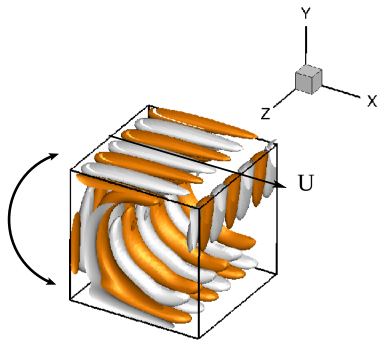
Figure 1. Schematic diagram of the open-cavity flow.



a) Time histories of the residual value R based on streamwise, normal and spanwise velocity components (U,V and W).

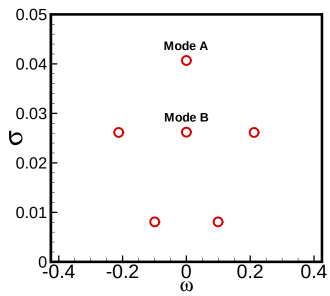


b) Spanwise velocity W at time 600

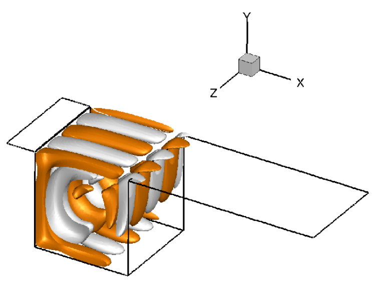


c) Spanwise velocity W at $Re_L = 4200$.

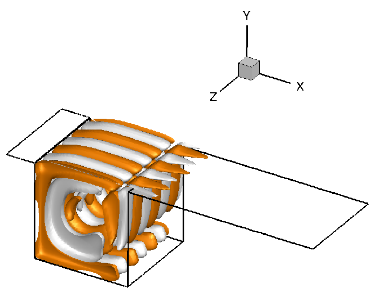
Figure 2. DNS analysis at $Re_L = 2500$. a) R is displayed in a logarithmic scale. An exponential growth regime is illustrated with a temporal amplification ≈ 0.0406 until saturation. b) The velocity is shown after saturation. The spanwise wavenumber $\beta = 2$ is selected. c) DNS analysis at $Re_L = 4200$. The spanwise wavenumber $\beta = 3$ is dominant. The regime is characterized by a slow beating of vortices inside the cavity.



a) Spectrum at $Re_L = 2500$. σ and ω denote the temporal amplification rate and the circular frequency respectively. The most unstable mode is stationnary with $\sigma = 0.0406$.



b) Mode A: spanwise velocity perturbation. $\beta = 2$ is selected.



c) Mode B: spanwise velocity perturbation. $\beta = 3$ is selected.

Figure 3. Three-dimensional global linear stability analysis at $Re_L = 2500$.