EFFECTS OF GRAVITY AND MAGNETIC FIELD ON THE ONSET OF THE AXIAL INSTABILITY IN TAYLOR-COUETTE FLOW SYSTEM.

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Summary The goal of this work is to study in an alternative way the effects of gravity field relative to the magnetic field imposed on the Taylor - Couette flow system.
Thus, in the case of an applied axial magnetic field, the influence of the latter leads to the delay in the appearance of the axial waves.
Otherwise in the case of a tilted flow system defined following an angle α with respect to the vertical axis, the influence of gravity has the effect to gradually disappear one by one the Taylor vortices (TVF) as α increases within the range $0°<\alpha<90°$ in the near of their appearance.
Under these conditions, we seek to achieve the theoretical prediction of these phenomena within the framework of the linear theory of stability using a variational method as Galerkin method which solves the eigenvalues problem; these data are compared with our measurements and theoretical values available in literature.

INTRODUCTION

In this paper, we propose to establish the system of equations governing the laminar basic flow by determining the three components associated with the mean velocity field. This is followed by consideration of the flow near the onset of the first instability (Taylor Vortex Flow). For this, we propose to approach this problem within a linear theory to establish the system of equations governing the stability of Taylor vortex flow in the small gap approximation. One mentions that this configuration constitutes the reference flow system used by many authors. It was chosen close enough to the experimental device adopted in order to compare the theoretical prediction to our measurements carried out by visualization (Fig1).

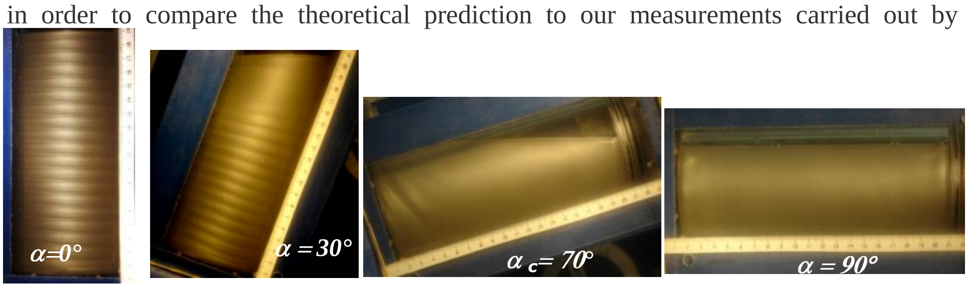

Fig1: Progressive relaminarization of the flow by titling the Taylor–Couette flow system near Ta=1849 (T_C=1697,4)

PROBLEM FORMULATION IN TITLED FLOW SYSTEM AND RESOLVING

The effect of the inclination angle α of the flow system introduces the influence of gravity acting on the axial and radial direction, hence the need to change the coordinate system. The transformation Cartesian coordinates *(x, y, z)* to the coordinates of intrinsic reference *(ξ, ζ, θ)* is proposed as follows $X = (\xi\cos\alpha + \zeta\sin\alpha)\cos\theta,\ Y = (\xi\cos\alpha + \zeta\sin\alpha)\sin\theta$ and $Z = \zeta\cos\alpha - \xi\sin\alpha$

The governing equations are based on the Navier–Stokes equations $\frac{\partial V}{\partial t} + V.\nabla V = -\frac{1}{\rho}\nabla p + \nu\Delta V$ and the continuity equation $\nabla\cdot V = 0$, satitisfying to the boundary conditions, $\xi = R_1/d : V_\xi = V_\zeta = 0, V_\theta = 1$ $\xi = R_2/d : V_\xi = V_\theta = V_\zeta = 0,$

$$\zeta = 0 : V_\xi = V_\theta = V_\zeta\ ,\ \zeta = \frac{H}{d} : \partial^2 V_\zeta / \partial\zeta^2 = 0$$

GRAVITY EFFECT AND LAMINAR FLOW SOLUTION

By performing the previous equations system in the small gap configuration $\delta = d/R_1 << 1$, we determine the three components of the mean velocity in laminar flow regime using the variable $\xi = x + R_1/d\ ,\ x = r - d/R_1$ in such a way $V_\xi = F(x)$ is a radial component, $V_\theta = G(x)$ a tangential component and $V_\zeta = \zeta H(x)$ is the axial component

where $F(x) = -\frac{1}{\mathrm{Re}}\frac{60}{61}x^2\ x-1^{\ 2}, G(x) = -\frac{61}{60}x + 1 + \frac{1}{60}\ 2x^6 - 6x^5 + 5x^4$ and $H(x) = \frac{1}{\mathrm{Re}}\frac{120}{61}x\ 2x-1\ \ x-1$

GRAVITY EFFECT AND TAYLOR VORTEX FLOW SOLUTION

The flow system considered is tilted with respect to z axis and we assume the following hypothesis as small gap configuration $\delta << 1$, aspect factor Γ is relatively large $\Gamma = H/d >> 1$, axisymetric flow $\partial/\partial\theta = 0$ and linear theory approximation. The expressions of the perturbation velocity field are as follow (Fig2).

$$\begin{cases} v_\xi = u(\xi).\cos(\lambda\zeta).\exp\sigma t, & v_\theta = v(\xi)\cos(\lambda\zeta)\exp\sigma t \\ v_\zeta = w(\xi)\sin(\lambda\zeta)\exp\sigma t, & p = p(\xi).\cos(\lambda\zeta).\exp\sigma t \end{cases}$$

By replacing the previous expressions into the governing system equations cited above we establish the stability equations system,

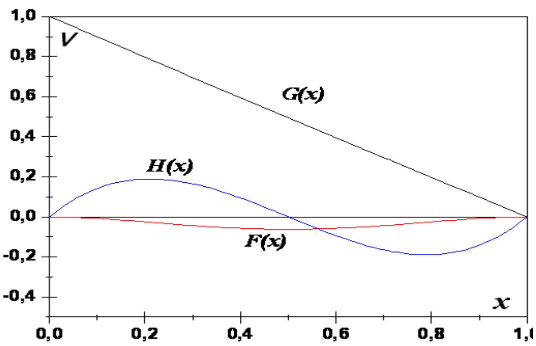

Fig2: Distribution of the mean velocity profile in the small gap configuration

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$LL_0 + \ell_1 \;\; u(x) = 2T_s\lambda^{*2}(\bar{V}_\theta / \cos\alpha).v(x)$ and $L + \ell_2 \;\; v(x) = D\bar{V}_\theta u(x)$ where

$D = d/dx$, $L = D^2 - \lambda^{*2} - \sigma$, $L_0 = D^2 - \lambda^{*2}$

$\ell_1 = \lambda^{*2}(\bar{V}_\xi D + D\bar{V}_\xi) - (\bar{V}_\xi + \lambda^{*2} D\bar{V}_\zeta)D - \lambda^{*2}(D^2+1)\bar{V}_\zeta$ and $\ell_2 = \bar{V}_\xi(1 + 1/(x + R_1/d)\cos\alpha)$,

the associated boundary conditions are u = v = 0 at x = 0 and x = 1

By using Galerkin method in order to seek a solution of the stability equations system as an eigenvalue problem we can easily achieve an approximation of the order n = m = 1. Therefore, we get the relationship of Taylor number versus dimensionless wavenumber λ^* corresponding to the marginal stability $\sigma = 0$

$$T_C = \frac{28}{27}\frac{(\lambda^{*2}\cos\alpha + 10)(\lambda^{*4}\cos^2\alpha + 24\lambda^{*2}\cos\alpha + 504)}{\lambda^{*2}\cdot\cos\alpha} \qquad \lambda_c^*(\alpha = 0°) = 3.117$$

and the critical value is then $\lambda_c^* = \dfrac{3.117}{\sqrt{\cos\alpha}}$ (Fig 3)

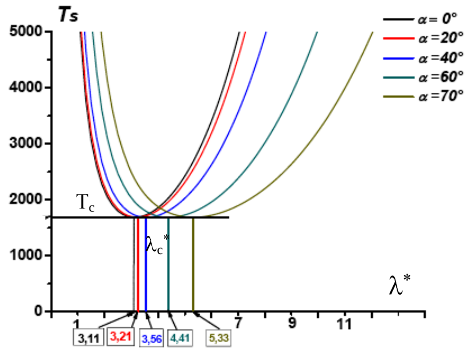


Fig 3: Diagram of marginal stability ($\sigma = 0$) near the onset of Taylor vortices

MAGNETIC FIELD EFFECTS

In this case the fluids between the two cylinders are conductors of electricity such as gallium which is applied an external magnetic field.

As S. Chandrasekhar we considered the problem in the case $\Gamma \to \infty$ and $d/R_1 << 1$ and axisymmetric flow.

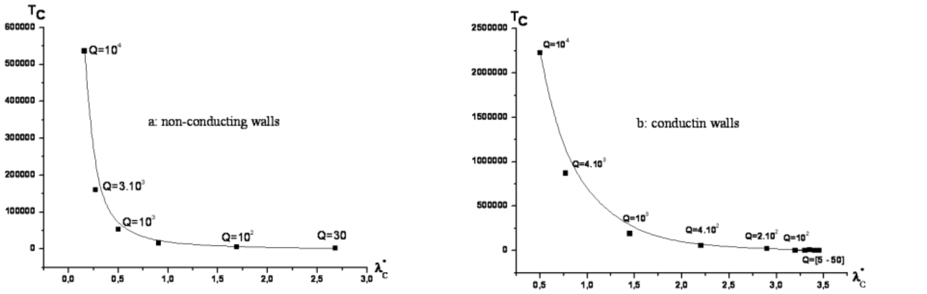
In the framework of linear theory we are looking for to determine critical Taylor number versus wavenumber for Chandrasekhar number $Q = \dfrac{\mu H^2 \sigma}{\rho\nu} d^2$ where Q plays the same role of Hartman number Ha ($Q = Ha^2$). The governing equations are

$$\begin{cases} \dfrac{\partial V}{\partial t} + V.\nabla V = -\dfrac{1}{\rho}\nabla p + \nu\Delta V + \dfrac{1}{\rho}\vec{J}\times\vec{B} & \nabla\cdot V = 0 \\ \dfrac{\partial B}{\partial t} = \nabla\times(V\times B) + \dfrac{1}{\mu\sigma}\Delta B & \nabla\cdot B = 0 \qquad \vec{j} = \sigma(\vec{E} + \vec{V}\times\vec{B}) \end{cases}$$

Where V, p, B, J are respectively the velocity, pressure, magnetic field and electrical density. The corresponding boundary conditions are $x = 0: V_x = V_z = 0, V_\theta = \Omega r, B_x = B_\theta = 0$ and $B_z = B_0$, $x = d: V_x = V_z = V_\theta = 0$

The effect of magnetic field on the critical number of axial wavelength λ_C^* is shown in Figure 4

Fig4: Evolution of the critical Taylor number as a function of wave number λ_C^* for (a) and (b) and different values of Q



For given Q, the calculations show that the critical Taylor number T_C decreases when the wavenumber λ_C^* increases in all cases (non-conducting and conducting walls). Thus, the effect of the magnetic field leads to delay the onset of axial instability and tends to modify the structure by significantly increasing the size of Taylor vortices.

CONCLUSION

Our main theoretical prediction shows that the effect of the gravitational field's role is to destroy the structure of the flow corresponding to the Taylor vortices near the onset at critical Taylor number $Tc_1 = 1750$ which is quite close to the experimental value $Tc_1 = 1708$. When the angle α increases corresponding to the inclination of the flow device, there is a gradual destruction of Taylor vortices and therefore, they lead to the relaminarization of the flow in laminar-turbulent transition regime. Thus, for fixed Ta we find that the critical wave number associated with the onset of vortices increase as α increases too. Regarding the effect of an applied axial magnetic field on the flow we see that the Taylor vortices are not destroyed but to delay their appearance and therefore stabilize the flow which is consistent with results obtained by S. Chandrasekhar. Conversely the wave number decreases as the applied magnetic field becomes more intense at large Chandrasekhar number Q.

In future work, we seek to extend the modelling of the combined effects of gravity and magnetic fields to assess their influences on the considered tilted flow system.

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