STABILITY OF QUASI-KEPLERIAN FLOW AT LARGE REYNOLDS NUMBERS

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Summary Stability of rotating shear flows with radially decreasing angular velocity but increasing angular momentum (quasi-Keplerian flow) is investigated experimentally and numerically at Reynolds numbers over a million. Such flows are known to be linearly stable at all Reynolds numbers but their nonlinear or subcritical stability is largely unknown. It is found that such flows are prone to be perturbed by nonoptimal axial boundaries which can generate significant secondary Ekman circulation. When Ekman circulation is minimized by independently rotatable rings at both axial ends, quasi-Keplerian flows are found to be robustly quiescent and strongly resilient against external non-axisymmetric perturbations. These results challenges our state-of-the-art understanding of nonlinear stability of shear flows and also provide important implications to angular momentum transport in astrophysical accretion disks.

INTRODUCTION

Quasi-Keplerian rotating shear flows are characterized by decreasing angular velocity Ω with radius, $\partial\Omega/\partial r < 0$, *and* increasing angular momentum $l = r^2\Omega$ with radius, $\partial l/\partial r > 0$. These include a special case of the so-called Keplerian flows where $\Omega \propto r^{-3/2}$ (so that $l \propto r^{1/2}$), as is the case for most thin accretion disks in astrophysics [1]. The Rayleigh linear stability criterion, $\partial l/\partial r \geq 0$, is satisfied in these Keplerian disks where vigorous turbulence is required to explain the observed fast accretion. Nonlinear instabilities in quasi-Keplerian flows are a candidate possibility if they can be found, especially at the large Reynolds numbers which are typical of accretion disks.

However, nonlinear instabilities have not been positively found thus far in quasi-Keplerian flows. Theoretically and numerically, numerous attempts have been made [e.g. 2]. It has been found numerically that the critical Reynolds number for nonlinear instabilities increases rapidly when the flow enters from the Rayleigh unstable regime to the quasi-Keplerian regime [3]. The first experimental study of quasi-Keplerian flows was performed by D. Richard [4] in 2001. (We note that the often-quoted [e.g. 5] experimental work by Wendt [6] and G.I. Taylor [7] were done in a different flow regime where $\partial\Omega/\partial r > 0$, and thus not directly relevant to the subject.) Both Richard and a more recent work [8] have claimed seeing turbulence in quasi-Keplerian flows. In contrast, we found that quasi-Keplerian flows are easily affected by nonoptimal axial boundary conditions which can generate secondary Ekman circulation and may explain the reported turbulence. Once Ekman circulation is minimized by independently rotatable rings, quasi-Keplerian flows are robustly quiescent [9] and strongly resilient against external perturbations [10]. In this presentation, we report the most up-to-date results on this subject mainly from experiments [9,10] at Princeton but also from a new numerical effort [11].

EXPERIMENTAL APPARATUS

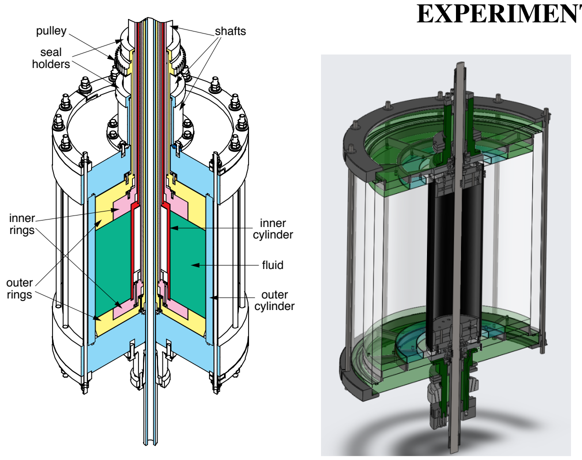

Figure 1. *Schematics of Princeton MRI Experiment when using water (left) and Hydrodynamic Turbulence Experiment (right).*

We first used the Princeton MRI (MagnetoRotational Instability) experiment [12] which is a short Taylor-Couette device with large gaps between cylinders so that $h/(r_2 - r_1) \approx 2$. To minimize Ekman circulation, the endcaps are divided into two, independently rotating rings [Fig. 1 (left)]. The dimensions are $r_1 = 7.1$ cm, $r_2 = 20.3$ cm, and $h = 27.9$ cm. The lower and upper rings are coupled. Currently, the MRI device is being used for the MHD experiment [13] with liquid metal and a stainless steel outer cylinder. The recently built HTX re-uses the acrylic outer cylinder from the hydrodynamic phase of the MRI apparatus. Each endcap is divided into three rings rather than two, but only the middle ring rotates independently, the other two being fixed to the inner and outer cylinders [Fig. 1 (right)]. The main diagnostic is a Laser Doppler velocimetry (LDV) system, which can provide synchronized measurements of two flow components at a given spatial point. Coincident measurements of radial and azimuthal velocities allow the Reynolds stress to be directly determined. To actively perturb the flow, we use an external pump to circulate fluid through a hollow inner cylinder axle to a set of four exhaust nozzles on the inner cylinder, with four intake nozzles for the return flow. Exhaust speeds upward of 1 m/sec are available. The nozzles are arrayed for $m = 2$ perturbations, with two sets of symmetric nozzles located at $1/4$ and $3/4$ of the inner cylinder height.

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RESULTS

We found that the quasi-Keplerian flows are extremely sensitive to axial boundary conditions which can be varied systematically by changing end ring speeds. Large changes occur in the bulk of the flow in (1) mean azimuthal speed profile across the radial gap, (2) fluctuation levels around the mean speeds, and (3) measured Reynolds stress [9]. These changes can be regarded as the changes in the secondary Ekman circulation driven by the endcaps [14]. By fine-tuning the end ring speeds, the ideal Couette profiles, $\Omega = A + B/r^2$ (solutions without axial end effects), can be approximated. In these cases, secondary circulations are minimized so that a large portion of the bulk flow near the mid height is free from the end effects when the Reynolds number is sufficiently large [9]. Fluctuation levels and Reynolds stresses are also minimized; in fact, both are indistinguishable from those measured in solid body flows which possesses no free energy for any instability. All the evidence points to robust quiescence of such flows without signatures of nonlinear instabilities at a shear Reynolds number as large as 2×10^6.

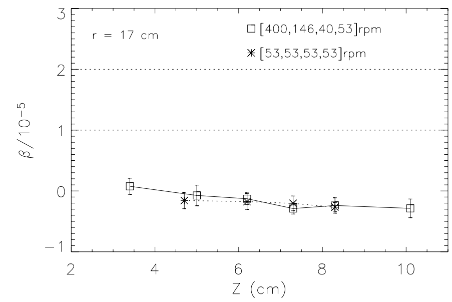

Figure 2. *The measured normalized Reynolds stresses as a function of height at a certain radius in a quasi-Keplerian flow at $Re \simeq 6 \times 10^5$ and a reference solid-body flow (from [9]).*

Optimized quasi-Keplerian flows are further studied through measuring their responses when actively perturbed in HTX [10]. Figure 3 shows a plot of the ratio of the fluctuation levels with and without active perturbations for Rayleigh unstable, quasi-Keplerian and solid body flows. These studies were performed with the nozzles configured so as to produce a dominant $m = 2$ azimuthal perturbation. The solid body cases show a larger response to the perturbations, especially at smaller Re. The decrease in the (normalized) fluctuations with increasing Re is expected since the perturbation magnitude is constant, whereas the mean flow increases. In contrast, the quasi-Keplerian and Rayleigh unstable flows show almost no change, suggesting that shear effectively suppresses the perturbations. These results indicate strong resilience of quasi-Keplerian flows against non-axisymmetric perturbations.

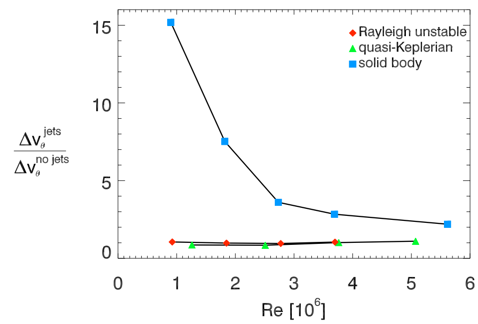

Figure 3. *Comparison of azimuthal velocity fluctuation levels with active perturbations normalized to the fluctuation level without active perturbations. In all cases there is negligible modification of the mean azimuthal velocity profile when the perturbation system is active.*

We also began to explore nonlinear stability of quasi-Keplerian flows at large Reynolds numbers using an efficient FFT-based incompressible code in a shearing box which approximates well local dynamics of quasi-Keplerian flows. The largest calculation we have yet performed with shearing-box boundary conditions is for $Re = 2 \times 10^6$ at resolution $N_x N_y N_z = 1024 \times 2048 \times 1024$. The results so far suggest that initially imposed turbulence still decays at these Reynolds numbers. Separately, we have also attempted viscous boundaries via some minor changes to the code but retaining the rectiliear geometry. Our largest viscously-driven flows to date have $Re = 1.3 \times 10^5$; sustained turbulence is seen near the joint between our simulated endcaps, qualitatively consistent with our experimental data [9] and also with the reported results by another group [8].

CONCLUSIONS

Stability of quasi-Keplerian flow is studied mainly experimentally and also numerically at Reynolds numbers over a million. It is found that such flows are extremely sensitive to axial boundaries, and when optimized, are robustly quiescent and resilient against perturbations. All evidence thus far points to absence of or extremely weak nonlinear instabilities if they even exist. These findings challenge our understanding of shear flow in general and provide important astrophysical implications in particular. Detailed and further updated results will be presented.

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