EXPERIMENTAL SCALING LAW FOR THE SUB-CRITICAL TRANSITION TO TURBULENCE IN PLANE POISEUILLE FLOW

Grégoire Lemoult[a)], Jean-Luc Aider & Jose Eduardo Wesfreid

PMMH (UMR7636 CNRS - ESPCI - UPMC Univ. Paris 6 - UPD Univ. Paris 7), École Supérieure de Physique et de Chimie Industrielles, 10 rue Vauquelin, 75005 Paris, France

Summary We present an experimental study of transition to turbulence in a plane Poiseuille flow. Using a well-controlled perturbation, we analyse the flow using extensive Particule Image Velocimetry and flow visualisation (using Laser Induced Fluorescence) measurements and use the deformation of the mean velocity profile as a criterion to characterize the state of the flow. From a large parametric study, four different states are defined depending on the values of the Reynolds number and the amplitude of the perturbation. We discuss the role of coherent structures, like hairpin vortices, in the transition. We find that the minimal amplitude of the perturbation triggering transition scales asymptotically like Re^{-1}.

INTRODUCTION AND CONTEXT

For more than a century, the transition to turbulence in shear flows has been a prolific domain of study [1]. Despite many theoretical investigations, it has not been possible to predict correctly this transitional process even for flows in simple geometries such as circular or plane Poiseuille flow (PPF), or plane Couette flow. Nowadays, the transition process in pipe and channel flows remains one of the most fundamental and practical problems still unsolved in fluid dynamics. Linear stability theory has been applied to PPF, linearizing the Navier-Stokes equation near the stable parabolic profile. The smallest unstable Reynolds number obtained was $Re_c = 5772$ [2]. Experimentally, one can found $1000 < Re_c < 2000$, highlighting the inadequacy of the linear stability theory to fully explain the transition to turbulence in PPF as in other shear flows.

Sub-critical transition to turbulence of shear flows, is characterized by an instability with respect to finite amplitude perturbation, where the larger the Reynolds number, the smaller the necessary perturbation. This behavior is described with a power law for the minimal amplitude of the disturbance triggering the transition: $\epsilon = O(Re^{\gamma})$. After the first studies of Trefethen *et al.*[4] on the critical exponent γ, Chapman studied the transient growth of sub-critical transition process in the PPF [5]. He found $\gamma = -3/2$ for initial streamwise vortices and $\gamma = -5/4$ for initial oblique vortices. On the other hand, the results of the nonlinear study of Waleffe and Wang show that $\gamma = -1$ in shear flows [6]. Critical exponents were measured experimentally in channel flow by Philip *et al.* [7]. They found, using a cross jet as disturbance, that the critical jet velocity triggering the appearance of hairpin vortices scales like $Re^{-3/2}$.

MATERIALS AND METHODS

The experimental system is composed of a 3 m long plexiglass channel. The test section's half height is $h = 10$mm, its length $220h$ and its width $15h$. The perturbation is generated $100h$ downstream from the inlet to ensure a fully developed Poiseuille flow for all Re. The x, y and z axis are respectively the streamwise, normal to the walls and spanwise coordinates, with $y = 0$ in the middle of the channel and $x = 0$ where perturbations are injected. The design of the inlet section, together with the smooth connections between all parts of the channel, minimize the upstream perturbations leading to a laminar base flow until at least $Re = 5500$ (The Reynolds number is estimated from volume flow measurements).

The flow is perturbed by *continuous* injection of water through four circular holes, normal to the flow, drilled into the the upper wall with diameter $d = 0.2h$ and spacing $\lambda = 3h$. The choice of a continuous perturbation, allows us to give a description of the statistical behaviour of the flow taking into account the spatio-temporal intermittency through time averaging. The structure of the flow induced by the jets may be complex and depends strongly on the amplitude of the perturbation defined as the velocity ratio $A = u_{jet}/u_{cl}$, where u_{jet} is the mean jet velocity and u_{cl} the unperturbed centerline velocity. In relation to the mass flow in the channel, our injection rate is $0.006A$. In our experiment, injection rate varies between 0.06% and 1%. For $0 < A < 2$, one can consider that each jet creates a pair of counter-rotating streamwise vortices. Those vortices generate through a lift-up mechanism some low and high speed streaks elongated in the streamwise direction, similar to those naturally present in coherent structures observed in transitional flow. It has been shown that the destabilization of those streaks is a key step in a self-sustaining process between streaks and streamwise vortices in Poiseuille flow [8].

The velocity field is studied using Particle Image Velocimetry (PIV) measurements. To take into account the spanwise modulation of the flow, we measure velocity fields in a volume ($\Delta x = 3h$, $\Delta z = \lambda = 3h$, $\Delta y = 2h$), centred around $x = 78h$. The measurement volume is divided in $11 x - z$ planes between $z = \pm\lambda/2$ around a jet. For each plane, 30 instantaneous snapshots are taken at 4Hz giving 11 time-averaged velocity fields. Then the PIV fields are space averaged over the streamwise direction. Finally, the 11 profiles are averaged along the spanwise direction giving the final mean velocity profile. Each mean profile presented in the following is the result of an average over 26000 profiles. The total time to measure one mean profile is 3 minutes, thus averaging intermittent effects.

[a)] Corresponding author. E-mail: gregoire.lemoult@espci.fr

DISCUSSION

The transition from the laminar to the turbulent regime is associated with a deformation of the mean velocity profile from a parabolic to a plug profile. The profile can then be used as a quantitative criterion to define the different states of the flow. We construct a state parameter $\tilde{u}$ as: $\tilde{u} = \frac{max(u)}{u_{cl}}$ where u is the perturbed mean velocity profile. We investigate the evolution of $\tilde{u}$ as a function of Re ($500 < Re < 5300$) and A ($0 < A < 2$). The state plane is sampled with 450 points. The sampling has been refined in the transition region. Two different domains can be clearly identified: the laminar and turbulent state, separated by a sharp cliff, giving a description of the global state of the flow. We now need to define a rigorous criterion to identify the transition.

For this purpose, fluorescent dye is added to the water tank used to feed the perturbing jets. Typical visualizations of the perturbations at $x = 20h$, near the exit of the jets, are shown on figure 1. For a small perturbation ($A = 0.167$), the flow is steady and laminar. Even if the fluorescent patches show traces of hairpin vortices, the mean velocity profile remains parabolic ($\tilde{u} = 1$). As the amplitude of the perturbation is increased ($A = 0.209$), hairpin vortices becomes larger but still smaller than the channel's half-height and slightly unsteady. In this case the mean velocity profile becomes asymmetric and $\tilde{u}$ decreases ($\tilde{u} = 0.9$). For $A = 0.248$, hairpin vortices becomes unstable, with intermittent excursions in the lower half of the channel. The velocity profile becomes flatter, more symmetric and $\tilde{u}$ still decreases ($\tilde{u} = 0.82$). Finally, for $A = 0.276$ (first row), the flow becomes turbulent with large mixing over the entire cross section, leading to a typical turbulent plug profile ($\tilde{u} = 0.8$).

Thanks to this analysis, it is now possible to define a quantitative criterion for the onset of turbulence. In the following, we will consider that the flow becomes turbulent when $\tilde{u} \approx 0.8$. For $0.8 \lesssim \tilde{u} < 1$, the flow is in an intermediate state dominated by hairpin vortices. We define the transition line as the destabilization of hairpin vortices, corresponding to $\tilde{u} = 0.81$. We show, in figure 2 a log-log plot of the minimal amplitude of the perturbation $A_c = (u_{jet}/u_{cl})_c = f(Re)$ triggering the transition, defined as $\tilde{u}(A_c) = 0.81$. The -1 slope proposed by Waleffe is added on the plot, showing good agreement for $Re > 2000$. For $Re < 2000$, the asymptotic regime is not reached and experimental points deviate from the -1 slope. Evaluating γ in a restricted range $1000 < Re < 2000$, leads to an underestimate value for the exponent close to $\gamma = -3/2$ as found by Cohen [7].

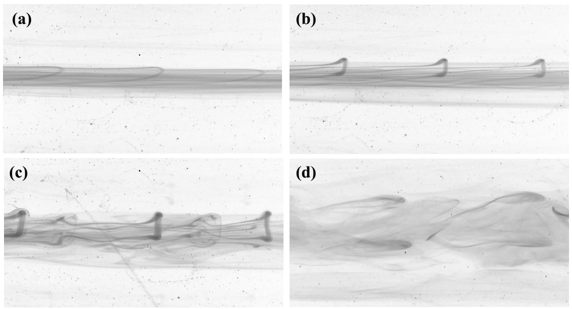

Figure 1. Upper side view using fluorescent dye at $x = 20h$, near the exit of the jets. A, b,c and d correspond respectively to $A =$ 0.276, 0.248, 0.209 and 0.167. Re is 2830 for all.

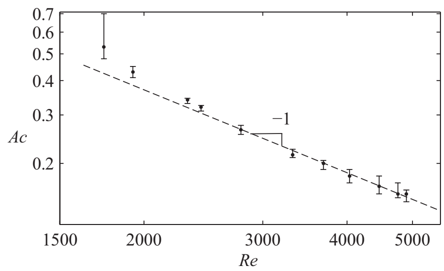

Figure 2. Log-log plot of the stability curve for the onset of turbulence in PPF. Points correspond to $\tilde{u} = 0.81$ and dashed line is the -1 slope. The minimal amplitude triggering the transition scales like Re^{-1} at higher Re.

CONCLUSIONS

The sub-critical transition of PPF has been studied quantitatively through extensive PIV and LIF measurements. A well-defined state variable, $\tilde{u}$, has been introduced. Thanks to a large number of $\tilde{u}$ measurements, we could define four different regimes of the flow. We then focused on the minimal amplitude of the perturbation, A_c, triggering the transition. We found that $A_c(Re)$ scales like Re^{-1} in agreement with the asymptotic non linear theoretical model proposed by Waleffe and Wang [6] for shear flows. Also, we contribute to develop new ways of experimental investigation to one of the most fundamental problem in fluid dynamics.

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