INTERMITTENT CHAOTIC MOTIONS IN PENETRATIVE CONVECTION

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Summary Convection in plane layer is considered for constant temperatures at the boundaries and quadratic dependency of density on temperature. Temperature interval comprise the point of maximum density, so unstable and stable layers may form and motions can penetrate to the stable layer from unstable one. There were shown the essential hydrodynamical differences and also dynamical differences (as considered in phase space) for time-periodic motions between classical Rayleigh-Bénard convection and penetrative convection for flows at large aspect ratios and absence of mean horizontal flow. A curious sequence of changes of regimes with growth of supercriticality was described including loss of hydrodynamical symmetry of "tails" of temperature profiles in periodic motions, subcritical transition to asynchronous doubly periodic and quasiperiodic motions and chaotic intermittency on the background of quasiperiodic motion.

Introduction

Generally penetrative convection can be defined as interaction of stable and unstable layers of fluid, when motions from unstable layers penetrate into stable layers, the review can be found in [1]. Here we consider interaction of statically stable and unstable layers due to nonlinear dependency of density on temperature. It is well known that for fresh water in the vicinity of 4°C density can be approximated quadratically as a function of temperature.
Mathematical description of such penetrative convection in plane layer was made in [2]. Steady and periodic motions were considered e.g. in [3] –[8]. Some experiments were described in [9]—[10]. Here we will consider fully nonlinear modes including time-periodic motions and transition to chaos.

Equations and the method of solution

The horizontal layer of water is confined between planes $z = 0$ and $z = h$. The boundaries are stress-free with constant temperatures $T_{\rm b}$ at the lower boundary $z = 0$ and $T_{\rm u}$ at the upper boundary $z = h$. It is supposed that $\min(T_{\rm b}, T_{\rm u}) < T_4 < \max(T_{\rm b}, T_{\rm u})$, then the point of density maximum which is taken approximately as $T_4 = 4$°C will be located inside the layer. The equation of state is taken as in [2] $\rho = \rho_4\,(1 - \alpha_4(T - T_4)^2)$ where ρ_4 is the density at 4°C, $T_4 = 4$°C, $\alpha_4 = 7.68 \cdot 10^{-6}\,(^\circ\mathrm{C})^{-2}$. The variables describing the motion are represented as the sum of functions in conductive state and the deviations from this state. The system of equations for deviations in the Boussinesq approximation can be written as follows:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\sigma\nabla p + \sigma\,\triangle\mathbf{v} + \sigma R\,T\,(T + 2\lambda - 2z)\,\mathbf{e}_{\rm z}, \tag{1}$$

$$\frac{\partial T}{\partial t} + (\mathbf{v}\cdot\nabla)\,T - w = \triangle T, \quad \mathrm{div}\mathbf{v} = 0 \tag{2}$$

where non-dimensional parameters are: $\sigma = \nu/\kappa$, $\quad R = g\alpha_4 h^3 (T_{\rm b} - T_{\rm u})^2/(\nu\kappa)$, $\quad \lambda = (T_{\rm b} - T_4)/(T_{\rm b} - T_{\rm u})$. Here σ is the Prandtl number, R is the Rayleigh number, λ specifies position of point of density maximum inside the layer. The dimensional parameters are: ν — the kinematic viscosity, κ — the coefficient of the thermal diffusivity, g — the gravitational acceleration.
Two-dimensional problem is examined. The periodicity cell with stress-free vertical boundaries is considered. The velocity components $\mathbf{v} = \{u,\,w\}$ and temperature T are expressed as truncated Fourier series with time-dependent coefficients. The Bulirsch–Stoer method [11] was used to solve the obtained equations. The nonlinear terms were computed by means of the pseudospectral method [12]. Libraries for Fast Fourier Transform (http://fftw.org) were used. The number of Fourier harmonics was up to 1024x256.

Periodic motions and sequence of bifurcations from static modes to chaos

We gave description of time-periodic motions that are stable in large aspect ratio layer. The length of the periodicity cell changes significantly with the increase of supercriticality: after transition from steady to periodic motion the horizontal scale is doubled for the solution to be stable in plane layer. It was shown that time-periodic motions occur through side to side oscillations near the upper boundary and not through circular motion inside the big vortices as in classical RB convection without mean flow. For studying transition to chaotic motion for penetrative convection the length of the periodicity cell was chosen to be the same as the length corresponding to stable time-periodic motion. In Fig. 1 the dependence of average Nusselt number $\mathrm{Nu_T} = \langle \partial T/\partial z \rangle_{z=0}$ on supercriticality is shown. With the increase of supercriticality there is a second Hopf bifurcation that is subcritical, but after this bifurcation the trajectory is initially periodic with doubled period. And only with subsequent increase of supercriticality the motion becomes quasiperiodic [13]. With further increase of supercriticality quasiperiodic motion loses stability through appearance of intermittent stochastic bursts, but unlike intermittencies described by Y. Pomeau and P. Manneville [14, 15] here the bursts are on the background of quasiperiodic

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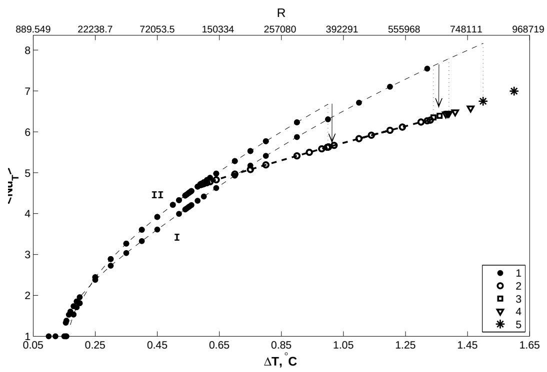

Figure 1. The dependence of the averaged Nusselt number on the supercriticality (temperature difference at the boundaries or R). 1, steady or conductive regime; 2, periodic; 3, doubly periodic; 4, quasiperiodic; 5, stochastic. I and II are the branches of hysteresis

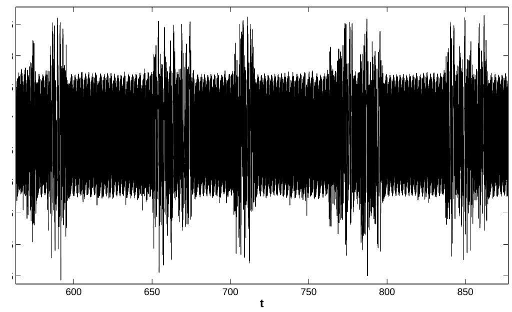

Figure 2. The dependence of the Nusselt number on time for intermittent mode. $T_{\mathrm{b}} = 4.75,\ T_{\mathrm{u}} = 3.25,\ M = 192,\ N = 96$

motions (Fig. 2). This intermittent mode becomes then unstable and window of quasiperiodicity is observed. After this window of quasiperiodicity another intermittent regime is formed with different frequencies.

Conclusions

Sequence of bifurcation with increase of supercriticality was shown for penetrative convection in case of density maximum to be in the middle of the temperature range between lower and upper boundaries. Qualitative hydrodynamical differences in periodic motion of penetrative convection and classical RB convection were described. The doubly periodic and quasiperiodic regimes were found. Stochastic intermittency modes on the background of quasiperiodic motion and its changing after passing through window of quasiperiodicity were investigated.

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