THE APPEARANCE OF HOMOCLINIC TANGLE IN PLANE COUETTE FLOW

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Summary In plane Couette flow a nonlinear time-periodic solution to the Navier–Stokes equations and its stable manifold have recently been found to form the basin boundary between laminar and turbulent flows. Here we present homoclinic orbits from/to this periodic solution. Through the Smale–Birkhoff theorem their existence explains turbulent bursting represented by vortical structures different from those in the familiar regeneration cycle. The homoclinic orbits are traced down to an onset Reynolds number for turbulent burst.

INTRODUCTION

Recent studies have shown that in transitional wall-bounded shear flows without linear instability of a laminar state the boundary between laminar and turbulent flows can be numerically identified in phase space as the edge of chaos [1]. A particularly interesting situation arises when the laminar-turbulent boundary is formed by the stable manifold of a traveling wave or periodic solution, which is then called an "edge state." Edge states have now been computed for flow in channels as well as pipes, with various numerical schemes and resolutions, and there is some consensus that they are a robust feature of subcritical shear flow.

Here we study the two-dimensional unstable manifold of an edge-state periodic solution [2, 3] to the incomressible Navier–Stokes equations in plane Couette flow, using a novel computational algorithm, and find that it contains two orbits which return to the edge state along its stable manifold. We conjecture, that these homoclinic orbits collide and disappear in a tangency bifurcation at lower Reynolds number. Away from this bifurcation, these orbits are transversal intersections of the stable and unstable manifolds. Thus, their presence generically implies the existence of an intricate tangle of these manifolds through the classical Smale–Birkhoff theorem. This, in turn, may explain the observed chaotic dynamics of irregular turbulent bursting.

NUMERICAL COMPUTATION

We consider plane Couette flow at a Reynolds number of $Re = 400$ in the minimal flow unit of dimensions $L_x \times 2h \times L_z$, where Re is based on half the velocity difference between the two walls, U, and half the wall separation, h. The streamwise and spanwise periods are $(L_x/h, L_z/h) = (1.755\pi, 1.2\pi)$ [4]. We used resolutions of $16\times33\times16$ as well as $32\times33\times32$ grid points in the streamwise (x), wall-normal (y) and spanwise (z) directions, respectively, and checked that the behaviour is qualitatively the same. The edge state in this computational domain is a time-periodic variation, which shows weak, meandering streamwise streaks [2, 3]. This gentle periodic orbit has a single unstable Floquet multiplier and thus its unstable manifold has dimension two and its stable manifold has codimension one in phase space.

A piece of the two-dimensional unstable manifold can be computed by Newton–Krylov continuation of orbit segments. A boundary value problem is set up which specifies that the initial point of the segment must lie in the linear approximation of the manifold, whereas the final point satisfies some scalar condition, e.g., the energy dissipation rate has a fixed value. This boundary value problem is under-determined by a single degree of freedom and thus we can compute a family of orbit segments by arclength continuation [5]. Since a connecting orbit separates the manifold into two components, the consecutive orbits segments can converge to a connecting orbit during the continuation.

HOMOCLINIC ORBITS

We have found two homoclinic orbits from/to the gentle periodic solution [6]. In Figure 1 one of the two homoclinic orbits is shown, which makes a large excursion during which the energy dissipation rate reaches more than twice the mean value in periodic motion and four times the value at the laminar equilibrium. In the dissipative phase of this bursting cycle, as shown in Figure 2, at the valley and the crest of the streaks small quasi-streamwise vortices are formed and move rapidly in the spanwise direction, and the larger part of the energy dissipation takes place there. Similar dissipative flow structures are also observed during bursting in a turbulent state, and they are very different from those for the near-wall regeneration cycle [4, 2].

The existence of orbits homoclinic to the edge state implies that the geometry of the laminar-turbulent boundary is rather complex and we can expect to find a manageable approximation to it only locally. At the same time, it generically implies the existence of infinitely many unstable periodic orbits which correspond to flows with arbitrarily many, arbitrarily long, near laminarization events. It is natural then to think of turbulent shear flow as governed by a large chaotic attractor which comprises both unstable periodic orbits in the turbulent regime, which reproduce the regeneration cycle [2], and ones which reproduce near-laminarization and bursting events.

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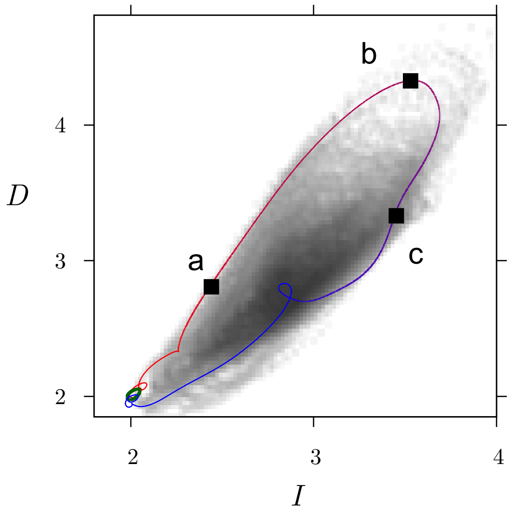

Figure 1. Projection of the orbit homoclinic to the gentle periodic solution (shown in green) in plane Couette flow at $Re = 400$ onto the energy input I and the dissipation rate D, normalised by their value in laminar flow. The piece of orbit leaving the gentle periodic solution is shown in red and the one approaching it in blue. In the background, the probability density function of turbulence is shown in gray scale. The labels **a**–**c** correspond to the snap shots in Figure 2.

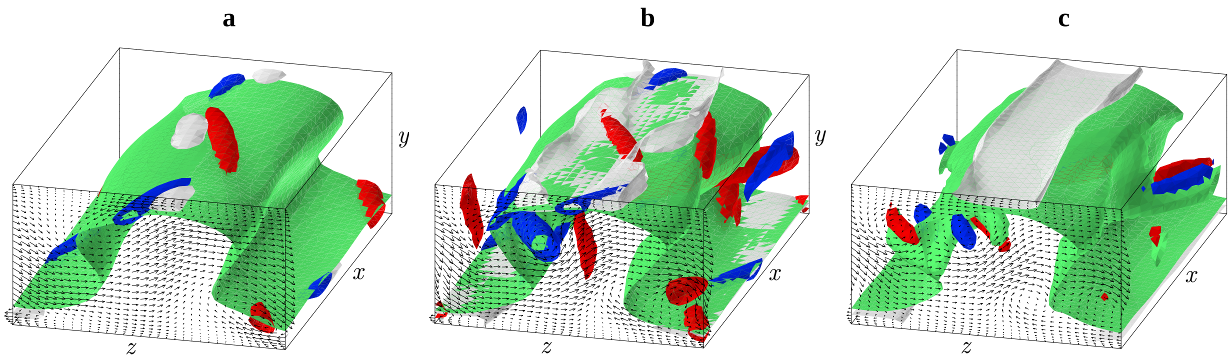

Figure 2. Visualizations of flow structures in one periodic box $L_x \times 2h \times L_z$ at three phases on the orbit homoclinic to the gentle periodic solution in plane Couette flow at $Re = 400$, labeled as in Figure 1. The green, corrugated isosurfaces of the null streamwise (x) velocity represent a streamwise streak. Red and blue objects are isosurfaces of $0.4\rho(U/h)^2$ for the Laplacian of the pressure, and denote the vortex tubes of the positive and negative streamwise-vorticity component. Gray isosurfaces show the local energy dissipation at 20 times the value of laminar dissipation. Cross-stream (y, z) velocity is also shown in the plane $x = 0$. The flow symmetry suggests that cross-stream velocity in the plane $x/h = L_x/2$ is given by the reflection of that in the plane $x = 0$ in the spanwise (z) direction.

APPEARANCE OF HOMOCLINIC ORBITS

As conjectured above, the two homoclinic orbits found here would collide and disappear at lower Reynolds number. The Reynolds number of such a tangency bifurcation could be considered to be an onset Reynolds number for a homoclinic tangle leading to turbulent burst, which should be significant for a theoretical interpretation of subcritical transition to turbulence in wall-bounded shear flow. In the presentation of this work, therefore, we shall also discuss the appearance of these homoclinic orbits.

References

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