SIMULATION OF MULTIPLE WAVY AND SPIRAL TAYLOR-GÖRTLER VORTICES IN CONCENTRIC SPHERICAL GAPS

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Summary The spherical Couette flow between two concentric spheres with only the inner sphere rotating is simulated using a fifth-order upwind compact finite difference method. Two medium-sized clearance ratios $\beta = (R_2 - R_1)/R_1 = 0.14$ and 0.18 are chosen to compare with literature. The spiral Taylor-Görtler (TG) vortex flow for $\beta = 0.14$ at $Re = 1110$ [Sha, J. Fluid Mech. **431**, 323 (2001)] was reproduced first. Then we study evolution of the 2-vortex flow with Reynolds number. it is found that the traveling waves develop at $Re = 1540$ will vanish at $Re = 6600$, where the flow returns to toroidal TG vortex flow, thus confirming the occurrence of a reverse Hopf bifurcation from the limit cycle to the fixed point (termed as "relaminarization phenomenon" [1]). The well known multiple flows for $\beta = 0.18$ from the first instability ($Re_{c_1} \approx 645$) to the onset of turbulent transition ($Re \approx 10000$) observed by Wimmer [2] are revisited. The simulation confirms Wimmer's observation that a periodic 2-vortex flow can coexist with the steady 0- and 1-vortex flows at low Re regime. The reverse Hopf bifurcation also occurs for this periodic 2-vortex flow with increasing Re number. The steady 0- and 2-vortex flows lose stability to wavy disturbances on TG vortices for $Re \geq 6500$, and to spiral disturbances in the secondary flow region for $Re \geq 7100$. Nevertheless, the steady 1-vortex flow is very stable and does not undergo any wavy disturbance up to $Re = 8500$, where it loses stability to multiple spiral disturbances. The computational results are in agreement with the viewpoint that several bifurcations are necessary in flow transition to turbulence in enclosed domain.

INTRODUCTION

We consider the spherical Couette flow (SCF) with the inner sphere rotating and the outer sphere stationary. Superficially, there are two control parameters that determine various flow regimes: a Reynolds number $Re = \Omega R_1^2/\nu$ and a clearance ratio $\beta = (R_2 - R_1)/R_1$, where ν is the kinematic viscosity, Ω is the angular velocity of the inner sphere, and R_1 and R_2 are the radii of the inner and outer spheres. Physically, other factors such as initial flow states, rotational acceleration rate, and types of imperfection presented may also affect the formation of disturbances when the finial parameters (Re, β) are located in regimes of bifurcation solutions. Extensive studies were conducted on SCF [1, 2]. Various types of flow patterns strongly depend on the clearance ratio β. The Taylor instability in the form of Taylor vortices occurs as the first instability for narrow gaps and medium gaps, while the cross-flow instability in the form of periodic spiral vortices and shear waves occurs as the first instability for wide gaps. An intermediate-gap regime is particularly delineated by Nakabayshi[1] as $\beta_N \approx 0.1 - 0.13 < \beta \leq \beta_I \approx 0.17$ for which the spiral TG vortices or traveling waves after the second instability will disappear with increasing Re (reverse Höpf bifurcation).

A well-known feature of SCF is that a variety of distinct higher flow modes can occur at the same supercritical Reynolds number, in addition to the existence of multiple steady-state Taylor vortex flows with different number of TG vortices at the same lower supercritical Re number. This is a typical dynamical behavior of solutions to the N-S equations. Previous experimental investigations have revealed that the formation of multiple flow modes depends on the initial flow mode and on the Reynolds number history approaching the final Re [2], a typical behavior of bifurcation solutions. Experiments have obtained multiple flow modes such as multiple Taylor vortex flows [2], multiple traveling waves on TG vortices [1], multiple shear waves with different waves number and rotational frequencies.

In related numerical studies, although multiple steady flow modes with axisymmetric toroidal TG vortices as well as many higher flow modes with spiral TG vortices and shear waves have been simulated, two experiment observations that periodic flows coexist with steady Taylor vortex flows at the same Reynolds number [2], and that periodic flows "relaminarlize" [1] (i.e. the disappearance of velocity fluctuation with increasing Re), have not received much attention. The present work make give numerical simulation of such phenomena in SCF for the first time.

NUMERICAL METHOD

The governing equations are the three-dimensional incompressible Navier-Stokes equations. They are solved with the artificial compressibility method combined with the dual-time stepping technique. The equations are linearized to previous pseudo-time level, and are approximated with upwind difference for the convective terms and central difference for the viscous terms in the left hand side. The LHS is approximately factorized and solved with diagonalized Beam-Warming scheme. The convective terms in the RHS are approximated with the flux-splitting-based fifth-order upwind compact scheme, and the viscous terms are discretized with the six-order central compact scheme. The resulting tri-diagonal system is solved with a parallel block SPP solver [3]. The mesh number used is $34 \times 512 \times 201$ in the radial, circumferentially, and azimuthal directions, respectively.

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NUMERICAL RESULTS

For the case with $\beta = 0.14$, we start from the steady 2-vortex flow at $Re = 1000$ and increase Re quasi-statically. The flow becomes a wavy TG vortex flow with 5 traveling waves on the TG vortices at $Re_{c_3} \approx 1540$. This periodic flow remains stable for $1540 \leq Re < 6600$, and it finally returns to a steady-state toroidal 2-vortex flow at $Re_{c_4} = 6600$, as show in Figure 1. The dimensionless rotational frequencies of the traveling azimuthal waves, defined as $\frac{f_w}{m f_1}$, are 0.451, 0.478, and 0.452, respectively, which are close to experimental data. The variation of the flow mode with Reynolds number shows that a periodic wavy TG vortex changes to a steady TG vortex. However, the wavy TG vortex flow between $Re = 1540$ and 6600 is essentially laminar periodic state, therefore, it would be better to call the transition as "reverse Höpf bifurcation" rather than "relaminarization".

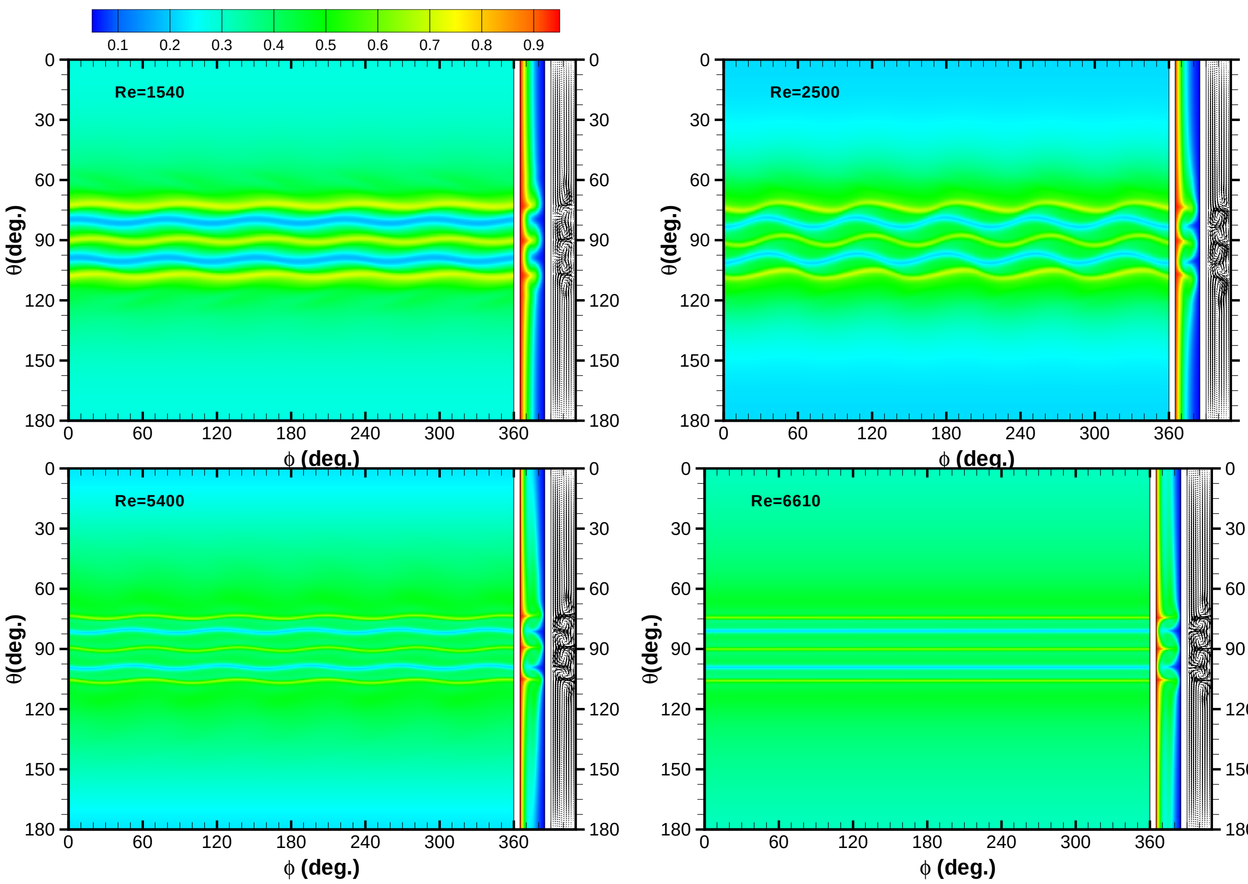

Figure 1. Variations of the wavy TG vortex flow ($N_T = 2, m = 5$) with increasing Re number. For each Re number, the left portion is instantaneous iso-values of the azimuthal angular velocity quantity ($\omega = v_\phi / r \sin\theta$) in the unwrapped middle spherical surface $r = (1+\beta)/2,\ 0 \leq \theta \leq \pi,\ 0 \leq \phi \leq 2\pi$, the middle graph is that in the meridional plane at $\phi = 2\pi$, and the right graph is velocity vectors (v_r, v_θ) in the meridional plane at $\phi = 2\pi$. The clearance ratio $\beta = 0.14$.

For the case with $\beta = 0.18$, coexistence of steady TG vortex flows and periodic flows at the same Re number was reported in [2] but has not been simulated before. We have successfully simulated the oscillating 2-vortex flow coexisting with steady 0- and 1-vortex flows. Similar to the case with $\beta = 0.14$, this oscillating 2-vortex flow gradually disappears as Re increases to 2250 where reverse bifurcation occurs. We have also simulated several new disturbances like multiple traveling waves and shear waves imposed on the same TG vortices by using a special form of perturbation.

References

[1] Nakabayashi K, Sha W M, and Tsuchida Y, "Relaminarization phenomena and external-disturbance effects in spherical Couette flow," J. Fluid Mech. **534**, 327–350 (2005).

[2] Wimmer M. Experiments on a viscous fluid flow between concentric rotating spheres. J. Fluid Mech., 1976, **78**:317-335.

[3] Yuan L, Guo H, and Yin Z H, "On optimal message vector length for block single parallel partition algorithm in a three-dimensional ADI solver," Applied Mathematics and Computation **215**, 2565–2577(2009).