THREE-DIMENSIONAL INSTABILITIES IN FLOW PAST A ROTATING CYLINDER

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Summary Flow past a spinning and translating circular cylinder is investigated. The Reynolds number, based on the free-stream speed and the diameter of the cylinder, is 200. The non dimensional rotation rate, α (ratio of the surface speed and free-stream speed) is varied between 0 and 5. The two-dimensional steady base flow for various α is computed. Linear stability analysis of the steady two-dimensional base flow is carried out for three-dimensional perturbations that are assumed to be periodic along the span. The results are also compared with Direct Numerical Simulations conducted for a cylinder of span $12D$, where D is the diameter of the cylinder. Very interesting flow patterns are observed. Karman vortex street, along with the 'Mode A' spanwise instability is observed for α less than 1.91. Centrifugal instabilities appear in the flow for $\alpha = 3.01$ and larger. For $\alpha = 4.5$, the vortex shedding re-appears. Steady flow is observed for $\alpha = 5.0$.

THE METHOD

A stabilized finite element method is utilized to carry out the computations. Time integration of the Navier-Stokes equations is carried out for various rotation rates. The linear stability analysis is also carried out. First the steady state solution for various rotation rates is computed. The stability of these states to perturbations of various spanwise wavelength is investigated. The perturbations are assumed to be of the form $\mathbf{u}'(x,y,z,t) = \hat{\mathbf{u}}(x,y)e^{i\beta z}e^{\lambda t}$. Here, the wavenumber of the disturbance is $\beta\,(= 2\pi/\lambda_z)$ and λ_z is the wavelength. λ is the eigenvalue of the fluid system and governs the stability of the base flow. In general, λ is complex and can be represented as: $\lambda = \lambda_r + i\lambda_i$ where, λ_r and λ_i are the real and imaginary parts, respectively. λ_r represents the growth rate; a positive λ_r leads to an instability. The subspace iteration method is used for solving the eigen value problem and locating the eignavalue with largest real part.

RESULTS

Direct time integration

Figure 1: Re = 200 flow past a rotating cylinder: isosurfaces of vorticity.

Figure 1 shows the results from the direct time integration of the equations for various values of α. The fully developed flow for each case is shown. The $\alpha = 0$ case corresponds to a non-rotating cylinder. The mode-A instability is observed in addition to the von Kármán shedding. The spanwise wavelength of the instability, is 4D. The Strouhal number based on the time variation of lift coefficient is 0.198. As in the $2D$ computations, no vortex shedding is observed for $\alpha = 2.0$ and 3.0. The flow achieves a 2D, steady state. The computations for $\alpha = 3.5$ lead to an interesting result. Unlike the 2D computations, the 3D computations exhibit spanwise centrifugal instabilities. The observed wavelength is $\lambda_z = 0.92D$. The flow, from 2D computations, for $\alpha = 4.5$ is associated with time-periodic vortex shedding. The 3D computations show that the vortex shedding is accompanied with a strong centrifugal instability. Also, the vortex shedding is aperiodic. The flow for $\alpha = 5.0$ is free of all instabilities; even the 3D computations lead to a 2D steady flow.

Linear Stability Analysis

The steady-state solutions for various values of rotation rates are tested for their stability. Computations are carried out for $3.0 \leq \alpha \leq 5.0$. Figure 2 shows the variation of the growth rate of most unstable mode with wave-number, β for various α. The flow exhibits centrifugal instabilities for $\alpha \geq 3.01$. These observations are consistent with the ones from

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the 3D DNS computations. For 3.0$\leq$ α $\leq$4.3, all the modes corresponding to $\lambda_{\mathrm{r,max}}$ are real. However, for $\alpha > 4.5$, for some values of β, rightmost $\lambda_{\mathrm{r,max}}$ corresponds to complex modes. For $\alpha \geq 4.8$ two steady states exist. They can be realized by starting from different initial guess/condition. For 2D perturbations (β = 0), the solution computed by increasing α and started with solutions obtained from 2D DNS at α = 0.0, is unstable. On the other hand, the solution computed by decreasing α, and started with solutions obtained from 2D DNS for α = 5.0, is stable. The solution which is stable to 2D perturbations (β = 0) remains stable for all values of considered β. The other solution, which is unstable to 2D perturbations, has positive real eigenvalues for a range of β. These results are also shown in Figure 2. Figure 3 shows the maximum growth rate and the corresponding span-wise wavelength of the eigenmode. It also shows some of the eigenmodes. The length of cylinder along the span, for visualization of the eigenmode is $12D$. The centrifugal nature of the instabilities is clearly observed.

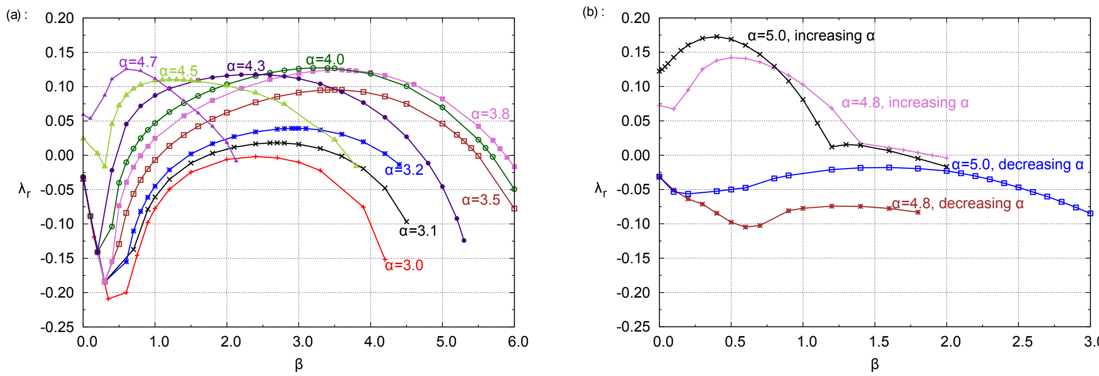

Figure 2: Re = 200 flow past a rotating cylinder: maximum growth rate, λ_{r}, for various wavenumbers, β and rotation rate, α.

ωx ωz ωx ωz

x

$\beta = 0.5$ $\beta = 1.4$

(a) Solutions obtained from increasing α and started from α = 0.0

ωx ωz ωx ωz

$\beta = 0.5$ $\beta = 1.2$

(b) Solutions obtained from decreasing α and started from α = 5.0

Figure 3: Re = 200, $\alpha = 4.8$: LSA for flow past a rotating cylinder : x and z component of vorticity field for the eigenmodes with maximum growth rate.

CONCLUSIONS

Three-dimensional instabilities in the flow past a spinning cylinder have been investigated. Two regimes of vortex shedding, for different α are observed. Centrifugal instabilities occur for $\alpha \geq 3.01$.

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