NUMERICAL SEARCH FOR THE HAIRPIN VORTEX SOLUTION IN MODERATE REYNOLDS NUMBER

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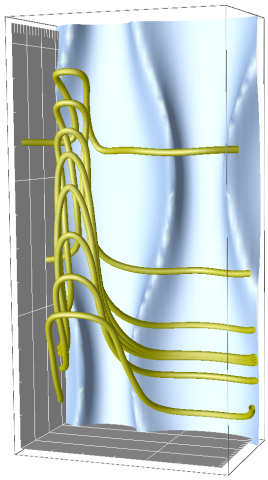
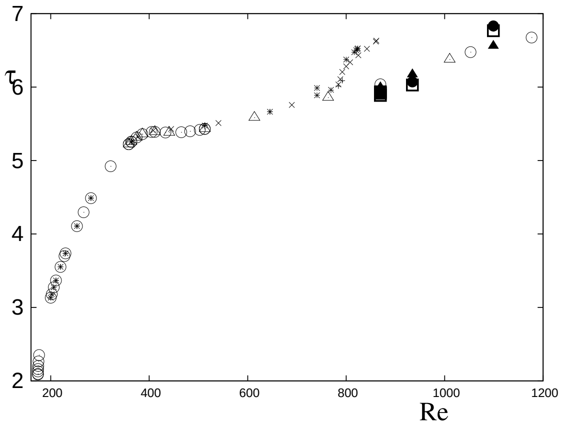
Summary An improved numerical continuation method is carried out to serach for the Hairpin Vortex solution (HVS, cf. Phys.Rev.Lett.102,114501 (2009)), an exact solution in plane Couette flow originally obtained at $Re \approx 200$. The implementation of higher precision arithmetic to the method enables us to trace the solution in higher Reynolds number values and also to obtain a more accurate spatial structure of the solution.



(a) The hairpin vortex solution (HVS) in plane Couette flow (PCF) at $Re = 200$. (Yellow) curves are vortex lines across the channel mid-plane, underneath which there are a couple of low-speed streaky structures visualised as (cyan) iso-surfaces.



(b) Bifurcation diagram of HVS in $Re - \tau$, where τ is the non-dimensionalized mean wall shear rate, $\tau = du/dz|_{z=-1}$. The different symbols correspond to different truncation levels (resolutions).

The Hairpin Vortex Solution (HVS) is an unstable solution in plane Couette flow solved recently by the authors, which contains vortex structures with the shape of a hairpin observed ubiquitously in turbulent shear flows. At the leg of the vortex structures, the localised vorticity lifts up the low-speed momentum fluid near the boundary so as to form a couple of low-speed streaky regions (see Fig.1(a)). At the same time, the head of vortex structures induces coalescence of these low-speed streak structures, which can then be visualised as streaky bulges beneath the head of the structure (not visualized here in Fig.1(a)).

By increasing the Reynolds number it becomes difficult to trace the upper branch of the HVS in previous numerical continuation. We carried out the continuation with several sets of truncation levels (different resolution) in double precision arithmetic. As shown in Fig.1(b), the upper branch of the HVS provides precipitous growth in the non-dimensionalized mean wall shear rate τ between $200 < Re < 400$, while the convergence of the branch is reduced over $Re = 600$. This makes the upper branch to appear apparently as a multivalued function. This fact could be related to the fact that coherent structures observed in turbulent shear flow become more complicated spatiotemporally with the increase of Reynolds number value.

CONCLUSION

In this work we improve the previous continuation method with multiple precision arithmetic. It is expected that the improved method will enable us to trace the solution precisely for higher Reynolds number values therefore obtaining a more accurate spatiotemporal resolution of the solution.

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References

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