STATISTICS OF GLOBAL FLOW TRANSITION IN ROTATING FLUIDS WITH FREE SURFACE

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Summary We have investigated statistics of global flow transition in rotating fluids with free surface, in the parameter range where temporary irregular surface switching between axi-symmetric and non-axi-symmetric shapes occurs with significant surface height variation. The statistical analyses elucidated that the event of the surface switching obeys a Poisson process, and the phenomenon can be represented by a superposition between the Poisson process and a quasi-periodic process.

INTRODUCTION

Rotating fluids accompanying with free surface shows variety of phenomena and typical bifurcation process in spite of its simplicity in the configuration (cf. Fig. 1 (A)). Rotating polygons of the free surface of flows in a cylinder driven by a bottom disk rotation is an example of the phenomena [1]. In case of relatively deeper fluid layers, the free surface shows interesting temporal transitions (termed 'temporally irregular surface switching' hereafter) [2]. Figure 1 (B) shows typical process of the surface switching that can be divided into symmetry breaking and symmetry recovery. In the symmetry breaking process the free surface having a round cross section changes its shape into an elliptical shape (Fig. 1 (B), (a)-(c)). Then the free surface detaches from the bottom of the cylinder (Fig. 1 (B), (c)-(d)). The detached free surface has humps at the bottom of the surface and repeats vertical oscillation. In the symmetry recovery process the elliptic free surface restores its axi-symmetry and reattaches to the bottom of the cylinder (Fig. 1 (B), (d)-(a)). The processes occur irregularly and thus some disturbances which have a distribution of the intensity may provide the irregularity. Qualitative and quantitative investigations of the flow field have clarified that laminar-turbulent switching (LTS) of the global flow accompanies the surface switching, namely the flow with the round free surface (cf. Fig. 1 (B), (a)) is laminar flow, and the flow with the elliptical free surface (cf. Fig. 1 (B), (d)) is turbulent flow [3, 4]. Here we discuss statistics of the global flow transition on time variations of the surface height.

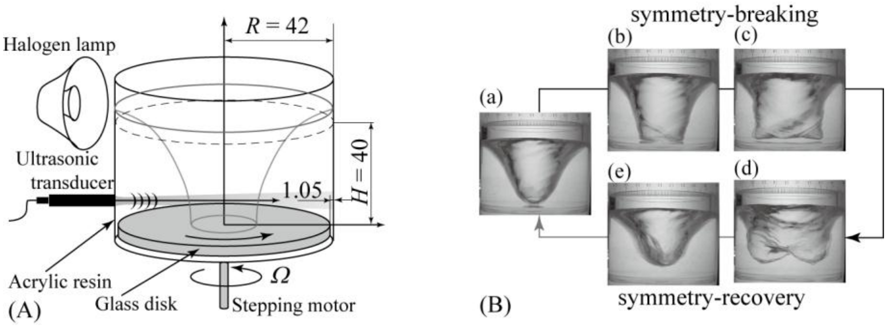

Figure 1 (A) Experimental setup, and (B) typical sequence of the switching process; (a) – (d) symmetry breaking and (d) – (a) symmetry recovery

EXPERIMENTAL SETUP

Experiments were done in the apparatus shown in Fig. 1 (A). Tap water in an acrylic cylinder with R = 42 mm in the inner radius is driven by rotation of a bottom disk. Height of the fluid layer is H = 40 mm at rest. The radius of rotating disk is smaller than the inner radius of the cylinder by 1.05 mm. Flow in this gap behaves like a Taylor-Couette flow [5], and wavy motions of Taylor vortices would provide local disturbance to induce the surface switching. Constant disc rotation with 800 rpm (13.33 Hz) was provided by a stepping motor, where the rotating frequency is somewhat larger than the critical rotating frequency for the onset of the surface switching [4]. Projection images of the surface shape illuminated by a halogen lamp were recorded by a digital video camera with 29.97 fps in the frame rate.

RESULTS AND DISCUSSIONS

Figure 2 shows a time variation of the free surface height, h, that is determined from a time-line image of pixels corresponding to the vertical center line of the cylindrical fluid layer on captured sequential images of the free surface. The variation seems quasi-periodic and its amplitude varies considerably with time, where large dips on the variation correspond to appearance of the surface switching. Figure 3 (a) shows the probability density function (PDF), p, of h

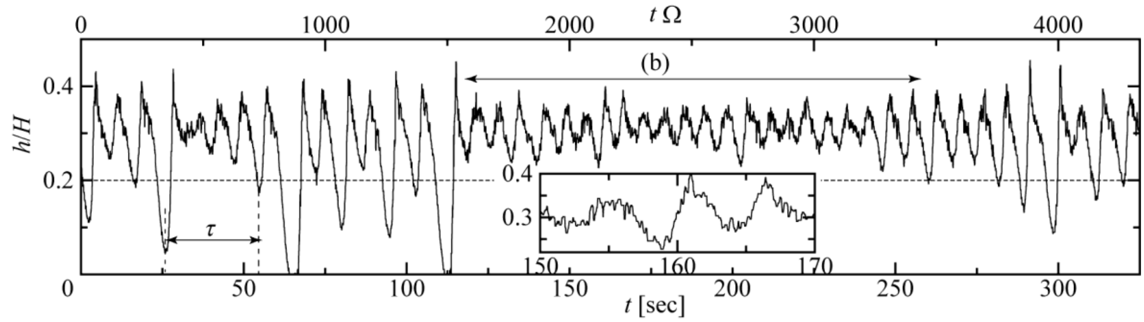

Figure 2 Time variation of the surface height, *h*, where the small figure inside the larger one displays enlarged part of the variation extracted from 150 to 170 sec

calculated from two sets of 600 sec time variation of *h*. The fitting curve in the figure represents a Gaussian function, $p(h/H) = A\exp[-(h/H - B)^2/C]$, where $A = 0.0731$, $B = 0.3131$, and $C = 0.0037$. The distribution is well represented by the curve in the range of $0.24 \leq h/H \leq 0.46$ that corresponds to the quasi-periodic part of the phenomenon. The left tail of the distribution, however, takes significantly larger values than the curve because of the appearance of the surface switching. Time scale of the quasi-periodic variation of *h* is given by a power spectrum of *h* as shown in Fig. 3 (b), where the corresponding time variation of *h* for the analysis is indicated in Fig. 2 as (b). The spectrum has a broad peak, $f_1 = 0.161$ Hz (6.21 sec), that corresponds to the quasi-periodic vertical oscillation. There is also a small peak in a higher frequency region, $f_2 = 5.15$ Hz (0.192 sec). The frequency f_2 is in the same order of the rotating frequency of the bottom disk, and is due to the humps on the rotating free surface (cf. Fig. 1 (B), (d)). Corresponding time variation of *h* for f_2 can be seen in the enlarged time variation shown in Fig. 2. To determine the stochastic characteristics of the surface switching, we investigate the PDF of time interval for appearing of the surface switching, $p(\tau)$, where large dips on h/H, which are smaller than 0.2, are assumed as the surface switching (see Fig. 2). The criterion was given by comparing between sequential images of the surface shape and the variation of *h* (see also Fig. 3(a)). Figure 3 (c) shows the cumulative distribution of $p(\tau)$,

$$P(\tau_1) = \int_{\tau_1}^{\infty} p(\tau)\, d\tau \cdot$$

The points of the distribution can be represented for $\tau_1 < 75$ sec by the relation, $P(\tau_1) = A\exp(-\tau_1/\tau_m)$, where $A = 22.44$, $\tau_m = 32.79$. This suggests that the appearance of the surface switching obeys a Poisson process, and the characteristic time of the interval is 32.79 sec.

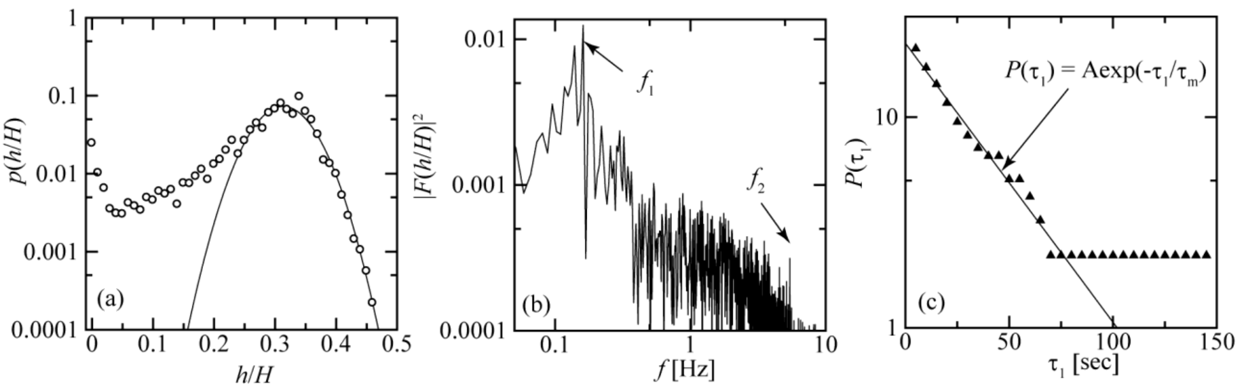

Figure 3 (a) probability density function of surface height variation, *h*, (b) power spectrum of *h*, where the corresponding data range of *h* for the analysis is indicated in Fig. 2, (c) cumulative distribution of probability density function of the time interval for appearing of surface switching

CONCLUSIONS

We investigated statistics of the global flow transition in the surface switching phenomenon on time variations of the surface height. The results indicated that the phenomenon can be divided into quasi-periodic process that can be expressed by main frequency f_1 and a stochastic process that obeys a Poisson process with a characteristic time τ_m.

References

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