HOMOTOPY OF EXACT COHERENT STRUCTURE BETWEEN PLANAR, CYLINDRICAL AND CIRCULAR CROSS-SECTIONAL SHEAR FLOWS: SINGLE AND DOUBLE LAYER STRUCTURE

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Summary We present a homotopy of nonlinear solutions from sliding Couette flow, i.e. flow between axially sliding concentric cylinders, to plane Couette flow and pipe flow by using special homotopy methods. The single/double layer solutions for sliding Couette flow are chosen as the starting point of the homotopy and it is found that the homotopy behaviors for these two solution types differ. This difference is explained by the shape of the critical layer and a forcing acting on the rolls.

INTRODUCTION

Nonlinear solutions of Navier-Stokes equations have been attracting as the key to understanding turbulence within the framework of a dynamical system theory. Such solutions are sometimes called exact coherent structures, due to their coherent quasi-streamwise rolls and streaks which are typically observed in shear flow transition. Paradoxically, most of the shear flows are linearly stable to streamwise rolls. So homotopy methods, Newton continuation transforming system, are widely used to obtain nontrivial solutions for shear flow instead of the usual nonlinear extension of linear instability [1][2][3][4][5]. Here we consider the known solutions of sliding Couette flow (SCF), which is the flow between axially sliding concentric cylinders to obtain solutions of plane Couette flow (PCF) and pipe flow (PF). Some special methods are developed to take the homotopy between planar, cylindrical and circular cross-sectional geometry. The single/double layer solutions for SCF [3] are used as the starting point of the homotopy and it is found that these two homotopy behaviors differ. The critical layer and the forcing acting on the rolls are examined to reveal the nature of this difference.

THE PCF-SCF-PF HOMOTOPY

We consider the full incompressible Navier-Stokes equations applied to a fluid in concentric cylinders. These equations are discretized by standard Chebyshev-collocaltion and Fourier-Galerkin technique. For this purpose, the cylindrical coordinate system is non-dimensionalized so that the radial interval [-1,1] represents the gap between the inner and outer wall. Assuming a travelling wave solution, the Newton method for spectral coefficients is used. SCF has two parameters, the Reynolds number, R, and radius ratio, η, and we recover PCF in the limit as $\eta \to 1$. Because we now fix the gap, it is reasonable to fix azimuthal periodicity in terms of length instead of angle, allowing non-integer values of the azimuthal wavenumber when η is changed. The new wavenumbers defined by length coincide with usual spanwise wavenumber at $\eta = 1$. PF is obtained in the limit as $\eta \to 0$ with proper axial pressure gradient. Note that the no-slip condition must still be applied in this limit on the infinitesimally thin inner cylinder. As a consequence, a homotopy with a special basis is introduced to eliminate the inner cylinder. We start from two types of mirror symmetric solutions of SCF [3]. Though they have the same symmetry, their spatial structures are different: one has large vortices whose scale is almost equal to the gap, whereas the other has double layered, half gap sized vortices. It is found that the homotopy of the double layer mode is relatively robust. As a result, we obtain the mirror symmetric solutions obtained by Itano & Generallis [4] for PCF and Pringle & Kerswell [5] for PF. However, the homotopy of the single layer mode is fragile. The resultant solution appears at values of the Reynolds number ten times higher than other finite amplitude solutions for PCF while we were unable to continue the single layer mode to PF.

THE FORCING AT CRITICAL LAYER: VORTEX WAVE INTERACTION

Here we refer to any component which is proportional to 0th/1st streamwise Fourier mode as “vortex”/“wave”. The “vortex” field is a streamwise averaged flow field, where the streamwise and cross-sectional components are called streak and rolls respectively. Fig. 1 and 2 shows the “vortex” component of velocity of lower branch solutions. We can see single/double layer property is preserved after homotopy. In the figures, the forcing on the “vortex” is also plotted. Here the forcing is defined by residual of streamwise averaged equations considering only the“vortex” component, and so the forcing originates from nonlinear terms. This forcing decays outside of the critical layer if the streak and rolls obey self-sustaining scaling [2]. However the “wave” component causes forcing on the “vortex” that smooths out the singularity inside the critical layer as classical asymptotic theory tells us. Indeed, we can see forcing is confined to the critical layer in the figures. We also confirm that the forcing from the present exact calculation agrees with the asymptotic expression derived in [6] along the critical layer at sufficiently high Reynolds number (Fig. 3). Surprisingly, it is found that the forcing is also localized along the critical layer in the vicinity of the streak. As vortex-wave interaction theory [6] tells there exists a jet at a tangential direction to the critical layer created by the forcing. The location and direction characteristics of the jets must occur to be consistent with the rolls and so determine the shape of the critical layer. Single layer type has a curved critical layer so that it gets close to the wall where jets are induced by two rolls and the wall. By contrast, the critical layer is always in the core region for the double layer solution, because in this case jets are induced between the vortex layers. As a consequent, the critical layer nearly perpendicular to the base flow gradient. This fact would explain robustness of this type of solution during the homotopy.

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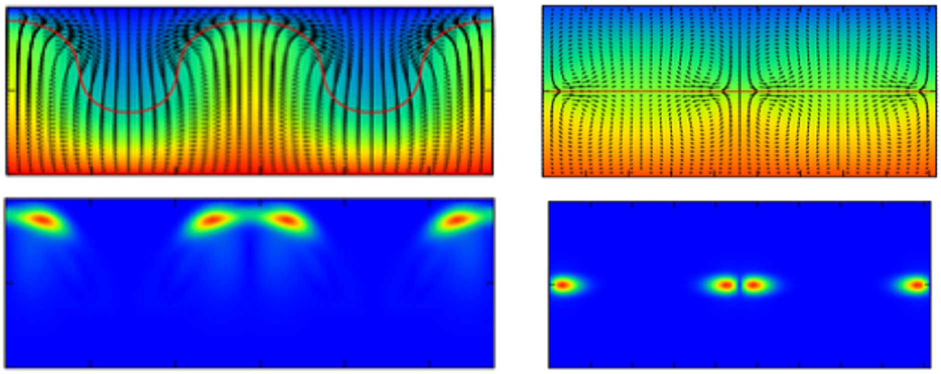

Figure 1. The “vortex” flow field (upper figures, red/blue represents fast/slow streaks) and the magnitude of the forcing acting on the “vortex” system (lower figures) of lower branch PCF solutions. Left: single layer mode, right: double layer mode. Red curves represent the critical layer.

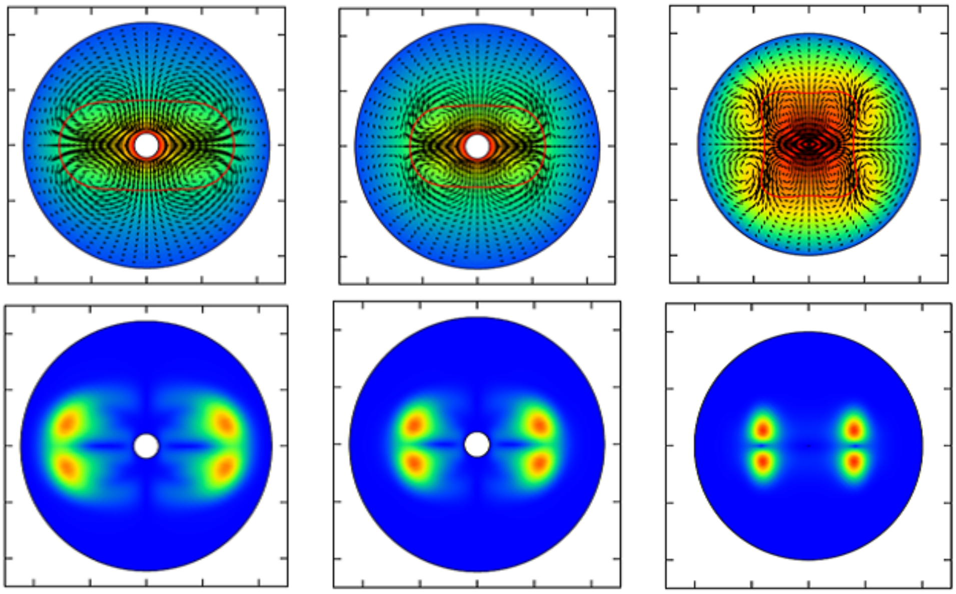

Figure 2. Same plot as Fig. 1 but for lower branch solutions of SCF and PF. Left: single layer mode of SCF, middle: double layer mode of SCF, right: double layer mode of PF.

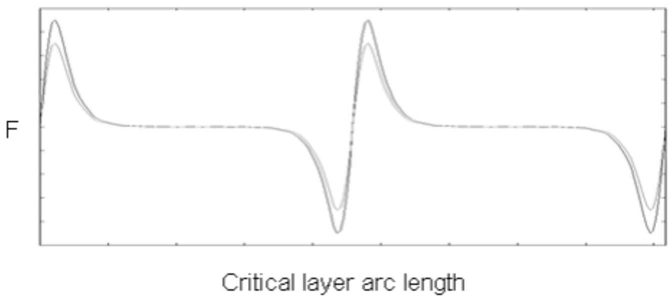

Figure 3. The forcing on the “vortex” system F at the critical layer for the double layer PCF solution. Dark gray: exact calculation, light gray: asymptotic calculation.

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