TURBULENCE TRANSITION IN THE ASYMPTOTIC SUCTION BOUNDARY LAYER

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Summary We study the turbulence transition in the asymptotic suction boundary layer (ASBL) by direct numerical simulation in small domains. Tracking the motion of trajectories intermediate between laminar and turbulent states we can identify the invariant object, the edge state. As in other cases, the edge state flow field is dominated by downstream vortices. It has the character of a traveling wave on short time scales, but also shows a secondary periodic variation on very long timescales. With the long-period oscillations comes a spanwise hopping by half the domain size. By embedding the flow in a family that connects the ASBL to plane Couette flow we can show that the slow modulation appears in a saddle-node-in-periodic-orbit (SNIPER) bifurcation.

INTRODUCTION

For intermediate Reynolds numbers, turbulence in the asymptotic suction boundary layer has to coexist with a laminar profile since the latter is still linearly stable (see [1] and references therein). Following the methods developed for internal shear flows [2, 3, 4, 5] we here analyze the boundary between laminar and turbulent dynamics in the asymptotic suction boundary layer. Using the lifetime as an indicator, we can identify the surface that separates smooth from chaotic variations of lifetime, the so-called *edge of chaos*.

SYSTEM AND ALGORITHMS

The asymptotic suction boundary layer (see figure 1) is the flow of a fluid in the vicinity of a porous plane into which the fluid flows with a perpendicular velocity V_0. This flow has a stationary exponential laminar profile solution for the downstream velocity that goes from 0 near the wall to U_0 far in the fluid, with the exponential decay thickness of $\delta = \nu/V_0$. By expressing the Reynolds number of the flow in terms of δ, one obtains $Re = U_0\delta/\nu = U_0/V_0$. Every point in the system's state space describes a divergence-free velocity field that fulfills the boundary conditions. By theorems on the uniqueness of solutions of differential equations every point considered as an initial condition $\vec{u}(t = 0)$ selects a unique trajectory that might decay directly towards the laminar state or reach turbulence. In both cases the initial conditions are located on opposite sides of the boundary separating laminar and turbulent dynamics.

By using a bisection method, the initial conditions that result in either laminar or turbulent dynamics can be brought arbitrarily close together within numerical precision. Starting from these conditions and performing further bisections whenever the orbits separate by more than a specified amount, one can trace the edge dynamics between laminar and turbulent flow for arbitrarily long times.

The numerical procedure for the calculation of fluid flows is based on the code Channelflow [6], which is designed for spectral numerical simulation of shear flows. The code was modified in order to allow for the suction/blowing boundary condition. We study a domain of dimensions $2\pi \times 4\pi \times 10$ (in units of δ) in the spanwise, downstream and wall-normal directions, with periodic boundary conditions in the spanwise and downstream directions, and with suction and blowing boundary conditions at the bottom wall and upper limit of the box, respectively. The numerical resolution used is $32 \times 32 \times 65$ points in each corresponding direction in the Fourier×Fourier×Chebyshev polynomial spectral representation. Varying the height of the domain we can also study the transition to plane Couette flow.

RESULTS

We study the system at a Reynolds number $Re = 400$. In order to characterize the turbulent intensity we use the kinetic energy of the spanwise and wall normal velocity components as a function of time as shown in the top right of figure 1. Over short time intervals, the flow behaves like a travelling wave dominated by ordered modulated streaks. On longer times one notes that the energy is not constant, as it would be for a travelling wave, but shows strong bursts at rather regular time intervals. During the bursts the streak structure is lost, but it reforms, translated in the spanwise direction by half the domain size, so that the total period of the oscillations is twice the distance between peaks.

Embedding the flow in a family of flows that interpolate between plane Couette flow and the ASBL, we find that the long period oscillations appear in a saddle-node-in-periodic-orbit (SNIPER) bifurcation. This is different from the bifurcation scenario suggested by [7], where the long period oscillations is linked to the the occurence of heteroclinic loops due to the $O(2)$ symmetry of the underlying flow.

Together with the large amplitude variations in energy come changes in the velocity field that are reminiscent of the streak breakdown phenomena observed in minimal flow units, [8]. During the slow phase with slowly increasing energy the flow

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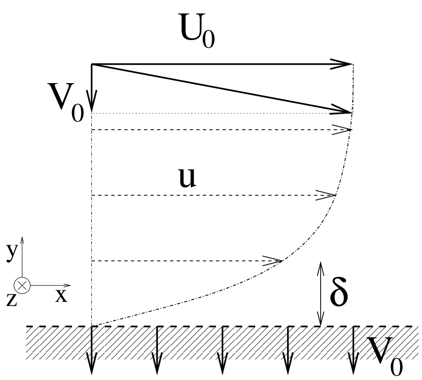
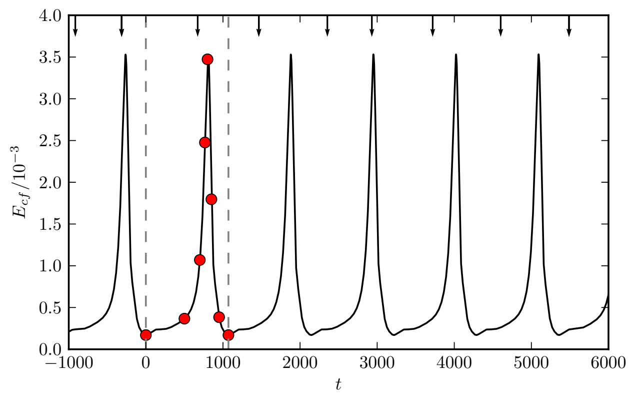
is dominated by downstream streaks which then break down and reform quickly once the flow is translated. These present observations are for edge states intermediate between laminar and turbulent, but they suggest that similar bifurcations can occur in the upper branch states that are part of the turbulent dynamics.





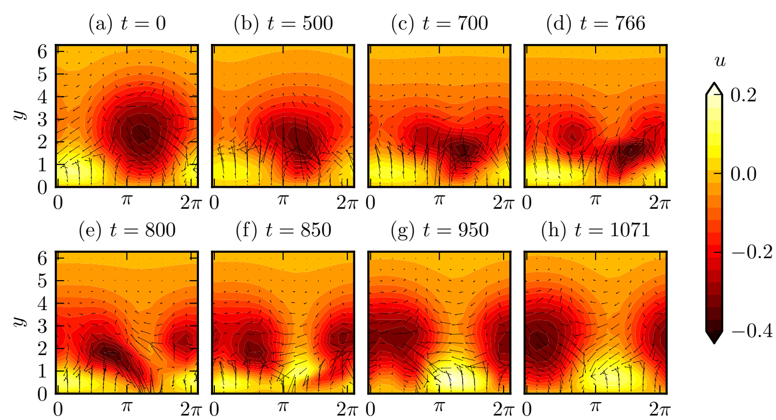


Figure 1. Top left: The geometry of the suction boundary layer flow. Top right: the energy in the y-z components for a trajectory tracing the edge state. The vertical bars mark the times at which the snapshots in the bottom row are computed.
Bottom row: Snapshots of the downstream component of the flow field in a $y - z$ cross section of the box for the times indicated in the top right figure.

References

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