WAVE PATTERNS ON FLUID INTERFACE SUBJECTED TO THE HORIZONTAL VIBRATIONS

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Summary Wave patterns on interface of two immiscible fluids of different densities subjected to the horizontal vibrations are studied. The case of two superposed infinitely long thin horizontal layers is considered analytically. The evolutionary system of differential equations is derived under shallow water approximation neglecting viscosity. One-parametric family of cnoidal solutions which are reduced to soliton-like solutions with the increase of wavelength is found. Stability of these solutions is studied. Analysis of the viscosity influence on the long-wave instability threshold shows that in the case when upper fluid is more viscous, viscosity makes destabilizing effect and vice versa. Direct numerical simulation of nonlinear dynamics of fluid interface in finite-length container is performed. Development of frozen wave and transition from this pattern to new quasi-stationary regime with the vibration intensity growth is observed. Wavelength of frozen wave is found to be non-monotonous function of vibration intensity with minimum in the transition point.

Vibrations exert significant influence on the behavior of mechanical systems. Even for simple mechanical system such as pendulum, vibrations can lead to the behavior qualitatively different than in the absence of vibrations. Vibration influence on hydrodynamical systems was studied for the first time in pioneer works by Faraday [1] and Rayleigh [2], where parametrically excited waves on fluid surface induced by vertical vibrations were considered. In the experiments by Wolf [3] the effect of horizontal vibrations on the behaviour of fluid interface was studied. It was found that in this case vibrations lead to the development of frozen wave at the interface. Theoretical analysis of the this phenomenon was carried out by Lyubimov & Cherepanov [4], who have developed average approach and implemented this approach for the investigation of instability of fluid interface under high frequency vibrations. They have shown that the development of wave patterns on fluid interface is related to the vibrational Kelvin-Helmholz instability and found that this instability is of long-wave type if $h < h_{cr} = \sqrt{3}$ where h is the dimensionless layer thickness measured in the capillary-gravitational units $L_{cap} \equiv \sqrt{\alpha/((\rho_2 - \rho_1)\,g)}$. At $h > h_{cr}$ the perturbations with finite wavelength are more dangerous.

In the first part of the present paper long-wave patterns on interface of two superposed infinitely long thin horizontal fluid layers subjected to the horizontal vibrations are studied. As well as in [4], the case of non-acoustic but high-frequency vibrations is considered and the vibration amplitude is assumed to be small. The case of layers of equal thicknesses is analyzed. The governing equations are derived in the framework of shallow water approximation based on the large ratio of horizontal dimension of hydrodynamical structures to the layer thickness. The evolutionary system of differential equations is derived in the framework of this approximation neglecting the viscosity. Stationary solutions of this system are obtained. It is found out that, additionally to the trivial solution with flat fluid interface, in subcritical range there exists one-parametric family of solutions with cnoidal profile of interface. With the increase of wavelength cnoidal profile is replaced by soliton-like profile. Additionally to that in supercritical parameter range another family of cnoidal regimes exists. Evolutionary equation for small perturbations is derived and linear stability analysis is made for all regimes. For trivial regime it is shown analytically that below critical value of the vibrational parameter it is stable, above critical value – unstable. Using analytical and numerical methods it is shown that cnoidal regimes are unstable.

In the second part of the paper the same configuration as in the first part is considered taking into account viscosity effects. For the description of viscous effects the multiple scale method is applied using boundary-layer spatial scale in vertical direction. Dynamical boundary layers near fluid interface and near rigid upper and lower external boundaries are taken into account.

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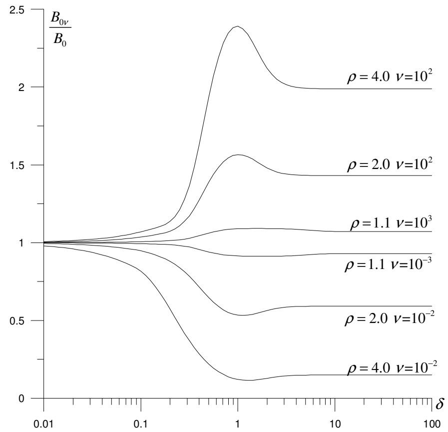

Fig.1. Viscosity effect on the long-wave instability threshold

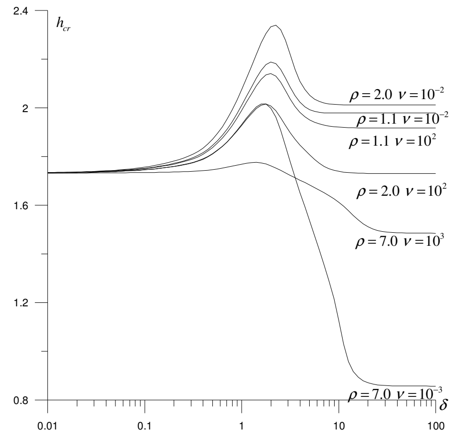

Fig.2. Viscosity effect on critical layer thickness at which long-wave instability is replaced by instability to finite wavelength perturbations

Critical value of vibrational parameter $B_{ov} \equiv B(k)\big|_{k=0}$ is determined for different density and viscosity ratios. It is found that, similar to the non-viscous case considered in [4], B_{ov} is nearly proportional to h. The ratio of B_{ov} to critical value of the vibrational parameter in the non-viscous theory B_o depends on three dimensionless parameters: $\rho \equiv \rho_2/\rho_1$, $\nu \equiv \nu_2/\nu_1$ and $\delta \equiv \sqrt{(\nu_1+\nu_2)\,\omega^{-1}}\Big/L_{cap}$. Besides, B_{ov} might be several times larger (or smaller) than B_o in the case when the viscosity ratio of upper and lower fluids is several orders of magnitude and the parameter δ is not too small. If upper fluid is more viscous then the viscosity makes destabilizing effect and vice versa (Fig.1). Depending on the parameters ν, δ and ρ the critical value of the layer thickness h_{cr} at which the long-wave instability is replaced by the instability to the finite wavelength perturbations (Fig.2). The asymptotic behaviour of the dependences $B_{ov}(\delta)/B_c$ and $h_{cr}(\delta)$ at large δ is studied analytically.

In the third part of paper direct numerical simulation of nonlinear dynamics of fluid interface in finite-length cavity is carried out for the case of large viscosity contrast. As calculations show, vibrations generate the waves propagating along the interface from the sidewalls. However, these waves are subjected to strong damping due to the viscous effects and at low intensity of vibrations the main part of the interface remain nearly flat but the velocity of fluid motion remains finite and the trajectories of fluid elements are nearly horizontal. When the dimensionless vibration velocity amplitude attains critical value, as well as in the case of a two-layer system of infinitely long horizontal layers of large enough thickness, frozen wave arises on the interface. As small supercriticalities the height of wave relief increases with the supercriticality according to the square root law and wavelength decreases. With further growth of the vibrational parameter transition from quasi-stationary wave pattern arising after stability loss by basic state with nearly flat interface to new quasi-stationary wave regime takes place and the decrease of wavelength with the vibrational parameter growth is replaced by the increase; the minimal value of wavelength corresponds to the transition point.

ACKNOWLEDGMENTS

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