TRANSIENT GROWTH OF PERTURBATION IN ROTATING POISEUILLE FLOW

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Summary The mechanism of transient growth of perturbation is explored. Energy growth of perturbation is computed over the full range of axial(k) and azimuthal(n) wavenumbers. For given Re and k, helical modes($|n| = 1$) undergo the largest volume-integrated energy growth through the resonance mechanism, and the imaginary part of the least stable eigenmodes is found to be very important to the levels of strength of the resonance. Meanwhile, the growth mechanism can be explained through the sign of uw, and $uw > 0$ shows the positive effect.

INTRODUCTION

Recently, transient growth of swirling flows has been greatly studied. One important phenomenon-the resonant phenomenon (with peaks of perturbation energy growth appeared at certain axial wavenumbers) for helical modes (with azimuthal wavenumber |n| = 1) in swirling flows has been discovered by Antkowiak & Brancher (2004) [1]. Then Pradeep & Hussain (2006) [2] gives more details about the physical mechanism of resonance, giving the conditions of the resonance. They have pointed out the resonance can be excited at every axial wavenumber k, but the reason why the largest energy growth located around a fixed k is not stated clearly. In this paper, we explain this reason in two ways, one is due to the effect of the eigenmodes, and the other is from the Reynolds-Orr equation.

METHOD

The cylindrical (r, θ, z) coordinates with corresponding velocity components (u, v,w) is adopted here. Lengths are scaled on the radius $r_{\max}$ and velocities are scaled on the pipe swirl $V_{\max}$. The Reynolds number based on these characteristic scales is $Re = V_{\max} r_{\max}/\nu$, where the ν denotes the kinematic viscosity. The mean flow is the rotating flow in a pipe given by

$$U = 0, V = \Omega r, W = 1 - r^2 \tag{1}$$

where the Ω is the rotation rate ration.
The perturbation is Fourier-decomposed along θ and z:

$$\{u, v, w, p\} = \{iF(r,t), G(r,t), H(r,t), P(r,t)\}\, e^{i(kz+n\theta)} \tag{2}$$

The volume integrated perturbation energy is

$$E(t) = \frac{1}{2}\int_0^\infty (|u|^2 + |v|^2 + |w|^2) r dr \tag{3}$$

The goal of the linear analysis is to determine optimal perturbations that maximize the energy amplification $E(t)/E(0)$ at time t.

$$G(t) = \max_{E(0)\neq 0} \frac{E(t)}{E(0)} \tag{4}$$

and $G_{\max} = \max_t G(t)$. The way to get the $G(t)$ can be referenced to Pradeep & Hussain (2006) [2]. We can get the same values as theirs by using the same mean flow.

RESULTS AND DISCUSSION

The effect of eigenmode for resonance

Figure 1a gives the maximum energy growth $G_{\max}$ as a function of the axial wavenumber k, and the peak can be easily found around at "$k = -0.1$". Antkowiak & Brancher (2004) [1] has pointed out the reason of the peaks in these similar figures should be the resonance, and Pradeep & Hussain (2006) [2] explained the resonance mechanism by using a simplified base flow: the top-hat vortex. This can be explained from the "Howard and Gupta" equation [3] that can be used to describe the equations in cylinder coordinates. The "Howard and Gupta" equation has singularity when $n\frac{V}{r} + kW - \lambda = 0$(we make $u(t) = \widehat{u}e^{-\lambda t}$ to get the eigenvalues). Pradeep & Hussain (2006) [2] have shown that when $\lambda_r = n\frac{V}{r} + kW$, the resonance will happen. Briefly, when rotation rate ration is the same to the frequency of one eigenmode(resonance

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mode), the resonance happens, but this cannot decide the level of the strength of the resonance. In fact, the singularity equation should be: $n\frac{V}{r} + kW - \lambda_r - i\lambda_i = 0$. The closer to the origin, the stronger the resonance should be. Therefore, the λ_i of the resonance mode is also important to the level of the strength of the resonance. Figure 1b gives the the grow rate of the least stable eigenmode as a function of the axial wavenumber k, and the location of the peak is around at "$k = -0.1$", which is the same to that in figure 1a. The same results can also be got by computing other parameters. This tells the location of the peak is decided mainly by imaginary part of the resonance mode.

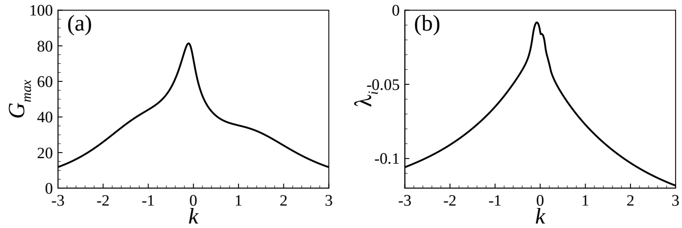

Figure 1. $Re = 1000$, $n = 1$ and $\Omega = 0.01$, (a) $G_{\max}$ vs. k, (b) Grow rate of the least stable mode vs. k.

The effect of Reynolds stress and growing time for resonance

The other way to explain the peak in figure 1a can be investigated from the Reynolds-Orr equation, given as

$$\frac{\partial E}{\partial t} = -\int_0^1 (uw\frac{\partial W}{\partial r} + uvr(\frac{V}{r})')rdr = -\int_0^1 (uw\frac{\partial W}{\partial r})rdr = \int_0^1 uw2r^2dr \quad (5)$$

where the production term $uw2r^2$ shows that positive Reynolds stress $uw > 0$ is necessary for energy growth. Figure 2a gives four different curves(different location in figure 1a) for the transient growth changes as time evolves. This can also tells that the energy growth for right-handed ($n/k < 0$) is larger than that for left-handed ($n/k > 0$). Figure 2b tells Reynolds stress changes as time evolves. For small waves, the grow rate($k = \pm 1$) is a little larger than that for long waves($k = \pm 0.1$). However, the growing time for small waves is less than that for long waves. The energy growth depends on both Reynolds stress and growing time(the area of each curve in figure 2b), so the area of the curve in figure 2b decides the value of the energy growth of the corresponding parameters. Then, it is easy to understand the curve for $k \approx -0.1$ has the largest energy growth because it has the biggest area.

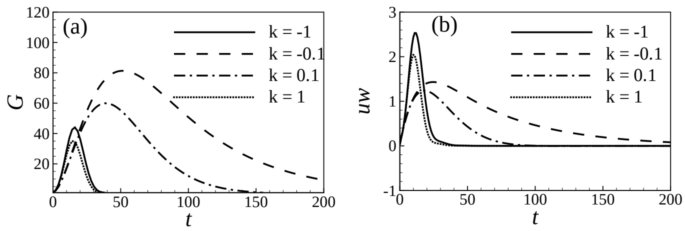

Figure 2. $Re = 1000$, $n = 1$, $\Omega = 0.01$ and $\Omega = 0.01$, (a) Energy growth G vs. t, (b) uw vs. t.

CONCLUSIONS

The mechanism of transient growth for the rotating flow in a pipe is discussed through two equations. One is from the "Howard and Gupta" equation, real part of which decides the onset of the resonance while the imaginary part of which decides the level of the strength of the resonance. The other equation is from the Reynolds-Orr equation, telling that the value of the energy growth depends on both the Reynolds stress uw and growing time t. The larger uw and the longer t result in larger energy growth.

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