STABILITY ANALYSIS OF COMPRESSIBLE SWIRLING JETS

Zhenhua Wan*, Lin Zhou *a), Dejun Sun*
*Department of Modern Mechanics, University of Science and Technology of China, Hefei, 230027, China.

Summary
In the present study we examine the viscous temporal linear stabilities of circular compressible swirling jets. The base flows that differ in swirl intensity are obtained numerically by solving axisymmetric boundary-layer equations in cylindrical coordinates. Eigenvalues and eigenfunctions of the linear stability equations are calculated by spectral collocation method. Two types of unstable disturbance modes are found for swirling jets, which are corresponding to Kelvin-Helmholtz (shear) instability and Rayleigh (centrifugal) instability, respectively. With the increase of swirl, secondary and higher unstable modes corresponding to centrifugal instability are observed, and amplification rates are substantially enhanced for both shear and centrifugal modes .

INTRODUCTION

Swirling effect is of great importance for a range of engineering applications such as combustion, flow control etc. It is well known that swirl can increase shear-layer spreading rate so as to enhance mixing of fuel/air in someway in combustion chambers. Under the enlightenment of Ref.[1], swirl might be utilized to suppress noise emission. Hence, the stability characteristics of a compressible swirling jet are deserved to be investigated in order to design efficient control techniques. The temporal linear stability of a compressible swirling jet is examined in Ref.[2]. The similarity solution of an axisymmetric incompressible jet with weak swirl is adopted as base flow, so only very weak swirling cases were considered in Ref.[2]. Currently, we particularly employ the numerical solution of the boundary-layer equations in cylindrical coordinates as the base flow which allow a jet with a comparatively high amount of swirl, the details of the equations and solving process are introduced in Ref.[3, 4]. In current study, the stability characteristics of swirl are explored in detail, and the results are compared with a swirling-free jet.

BASE FLOWS AND VALIDATION

The base flow is the numerical solution of compressible boundary layer equation in cylindrical coordinates. The central quantities of jet are utilized for nondimensionalization of the equations, Mach number ($Ma = w_c/a_c$), Reynolds number ($\mathrm{Re} = \rho_c w_c \delta_\omega/\mu_c$), where w_c is the central velocity, a_c is the speed of sound, δ_ω is the vorticity thickness. Three main cases that differ in swirl intensity are chosen, parameters are listed in table 1, where w_e represents the coflow velocity, v_{max} is the maximum swirl velocity, δ_s is the swirling thickness, and their definition can be found in Ref.[4].

Table 1. Parameters of base flow cases chosen for stability analysis

Case	Mach	Re	$\bar{v}_{max}$	δ_ω	δ_s	w_e
J	0.8	500	-	1.89115	-	0.1
SJ1	0.8	500	0.104684	1.89528	1.91119	0.1
SJ2	0.8	500	0.433987	1.89412	1.84996	0.1

Table 2. Convergence properties of temporal eigenvalues $\omega = \omega_r + i\omega_i$

N	Ref.[4]	Present
22	0.95036-0.02307i	0.9503597391-0.0230697970i
64	0.9504813967-0.0231707958i	0.9504813965-0.0231707958i

For stability analysis, the flow variables are decomposed into a base-flow part and a perturbation, expressed as

$$q(r,\theta,z,t) = \bar{q}(r) + \widetilde{q}(r,\theta,z,t) \tag{1}$$

where the perturbation part can be further Fourier decomposed into $\widetilde{q}(r,\theta,z,t) = \hat{q}(r)e^{i(\alpha z + n\theta - \omega t)}$. The stability solver is validated by computing temporal eigenvalue of the first mode for poiseuille flow in a pipe $Re = 9600$, $\alpha = 1$, and $n = 1$, as stated in table 2.

RESULTS AND DISCUSSIONS

The eigenfunction amplitude $|\hat{w}|$ (figure 1) and the amplification rate ω_i (figure 2) of cases J, SJ1 ,and SJ2 are compared, which show the influence of the swirl intensity. Generally, the axial shear will lead to Kelvin-Helmholtz type (shear) instability, which is typical for free shear flow such as jet or wake. Therefore, shear instability plays a dominant role in all cases. A important distinction between cases with and without a certain swirl is that secondary or high order unstable

a) Corresponding author. E-mail: zhoulang@ustc.edu. Sponsored by the National NSF under Grant No.11072238 and the 111 project of China under Grant No.B07033.

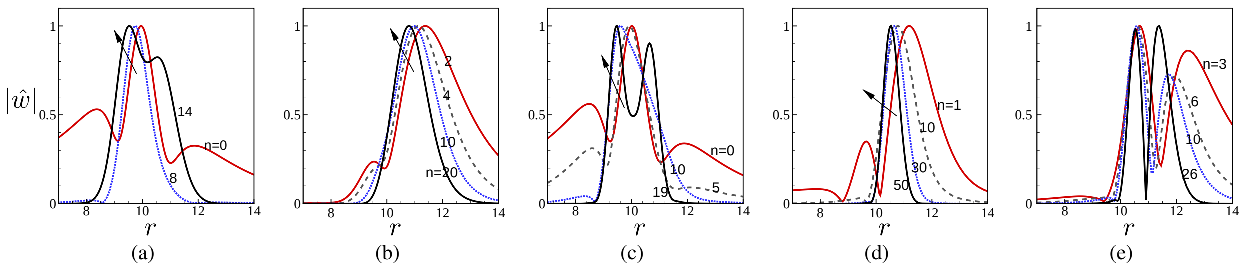

Figure 1. Eigenfunction amplitudes of streamwise velocity with a weak coflow for different azimuthal wavenumber (n); (a) shear mode (SJ1); (b) centrifugal mode (SJ1) ; (c) shear mode (SJ2); (d) first centrifugal mode (SJ2); (e) second centrifugal mode (SJ2).

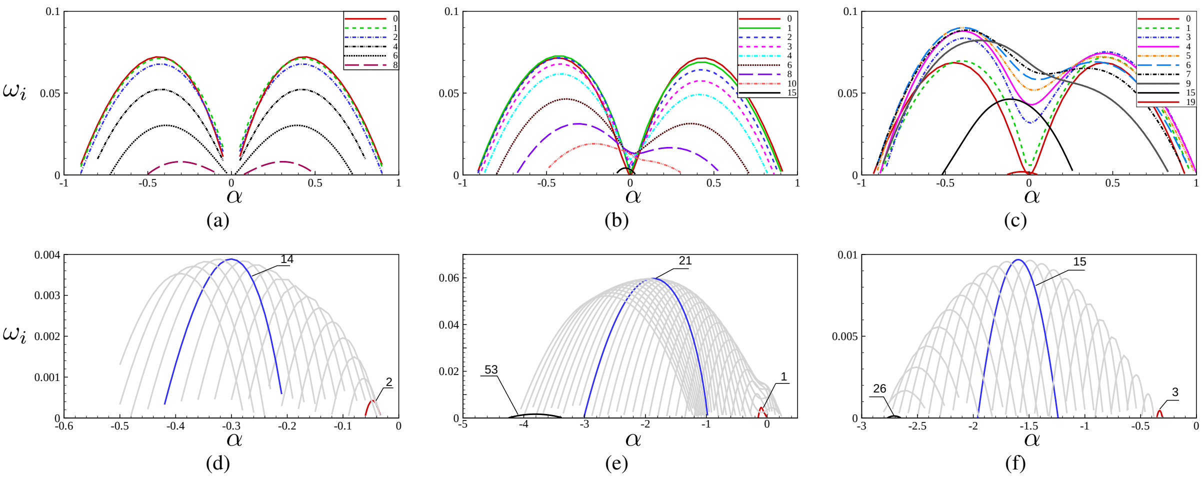

Figure 2. Amplification rate ω_i as a function of wavenumber α for different azimuthal wavenumber n. (a) shear mode (J); (b) shear mode (SJ1); (c) shear mode (SJ2); (d) centrifugal mode (SJ1); (e) first centrifugal mode (SJ2); (f) second centrifugal mode (SJ2).

mode appear to be the Rayleigh type (centrifugal) instability, which has not been stated in Ref.[2]. To be specific, one unstable centrifugal mode is observed in SJ1 and two unstable centrifugal modes are found in SJ2. As depicted in figure 1 (b), (d) and (e), the eigenfunction amplitude of streamwise velocity $|\hat{w}|$ of centrifugal modes deviate from the location where maximum shear occurs($r \approx 10$). For shear instability modes, figure 2(a) demonstrates that the axisymmetric $n = 0$ mode has the maximum amplification rate ω_i in the swirling-free case J, ω_i decreases as n increases, and the axial wavenumber $\alpha = 0$ disturbance is stable. After a certain of swirl is imposed (case SJ1 and SJ2), the $\alpha = 0$ disturbance becomes unstable for $n > 0$ unstable modes, and the amplification rate ω_i is firstly increased then deceased as n increases, which are in accord with the observations in spatial instability analysis[3, 4]. To compare figure 2 (a), (b) and (c), we can find following points: firstly, disturbances within negative α range become more unstable than positive α as $n > 0$ for swirling jets, and the two peaks of ω_i merge finally in the vicinity of $\alpha = 0$ as n is increased. Secondly, the increase of swirl results in many more unstable modes, specifically, there exist 15 unstable modes in SJ1 and 19 in SJ2. Finally, the most unstable mode shifts to a higher azimuthal wavenumber n as swirl is strengthened. As demonstrated in figure 2(f), there is a significant increase in amplification rate for the first several unstable modes, and the most unstable mode occurs at $n = 6$ for negative axial wavenumbers in SJ2. In addition to shear instability modes, the increase of swirl alters characteristics of centrifugal instability modes greatly as well. All the centrifugal unstable modes are generally confined to negative α for positive n, the region of instability for axial wavenumber space is significantly broadened, as shown in figure 2(d-f), and the peak of ω_i of unstable modes will rise first and fall later as n increases. As illustrated in figure 2(e), the most unstable mode occurs at $n = 21$ in SJ2, and the $n > 52$ disturbances become stable. More importantly, with swirl intensity increasing, high α disturbances becomes highly unstable, their amplification rates are substantially enhanced. In case SJ2, the maximum amplification rate ($\omega_i \approx 0.06$) for these centrifugal modes almost can be comparable to the corresponding value for the shear modes. Even for the second centrifugal mode in SJ2, the ω_i of $n = 15$ mode is two times higher than the maximum ω_i for centrifugal modes in SJ1.

The influence of heating, viscosity, the increasing of mach number and coflow will be further investigated in the following work.

References

[1] E. Laurendeau, P. Jordan, J. Delville and J.-P. Bonnet: *International Journal of Aeroacoustics* **7**:41-68, 2008.
[2] Mehdi R. Khorrami: *AIAA J* **33(4)**: 650-658, 1995.
[3] G. Lu, S.K. Lele : *Phys. Fluids* **11**: 450, 1999.
[4] Sebastian B. Müller, L. Kleiser : *Phys. Fluids* **20**: 114103, 2008.