MAGNETIC FIELD EFFECTS ON THREE DIMENSIONAL STABILITY OF NATURAL CONVECTION FLOWS IN DIFFERENTIALLY HEATED CAVITIES

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Summary The effect of increasing Hartmann number in generating quasi two dimensional structures with the vortices aligned with the magnetic field is investigated, in the context of Rayleigh-Benard convection. The scaling of Gr_{Cr} for the onset of steady convection with increasing Ha is identified. As the emerging steady convection rolls interact they lose stability subject to three dimensional travelling waves. Relevance with available experimental observations is discussed.

INTRODUCTION

The concept of Helium Cooled Lithium Lead (HCLL) blanket has been proposed for the design of ITER blanket. It consists of a number of rectangular boxes in which liquid lithium, mainly used for breeding tritium, flows via mild forced and free convection [1]. The above flow arrangement takes place under conditions of very large magnetic forces, high Hartmann number, and intense free convection, large Grashof number, owing to the very large volumetric heat production rate as a result of the fusion reaction in the reactor core. Free convection of Lithium inside rectangular boxes was studied numerically/asymptotically in [2] in the context of several flow arrangements, assuming steady state, i.e. vanishing or small Pe, and varying heat production rate Q(z) along the longitudinal direction, z, of the box. The assumption of negligible convection was based on the ratio $Gr/Ha^{5/2}$ being small. In was thus seen, accounting for the electric conductivity of the walls, that steady recirculating cells are produced, aligned to the magnetic field, owing to the temperature difference between the two opposing walls in the longitudinal direction.
In an effort to identify the scaling of Gr_{Cr} with increasing Ha and ascertain the validity of steady convection an experimental study was carried out regarding the onset of free convection in a rectangular cavity with a constant heat flux supplied through the bottom wall and a horizontal magnetic field [3]. The evolution of critical Gr was measured with increasing Ha, in the range of Ha~1000, for a box with side walls of large electric conductivity. The emerging vortices were seen to progressively assume a quasi 2d structure aligned with the direction of the magnetic field, fig 1a, especially as the magnitude of the magnetic field increased. Stability analysis incorporating the quasi 2d assumption was also performed and verified the tendency of Gr_{Cr} for steady convection to increase with increasing Ha, but cannot capture the experimentally observed onset of time dependent convection as Gr was slightly increased beyond Gr_{Cr}. The 3d stability analysis presented herein aims at capturing the scaling of Gr_{Cr} vs. Ha for steady convection, identify the nature of the ensuing transient convection and interrogate the range of validity of the quasi 2d assumption. The above issues are essential to understanding the dynamics of transition of free convection flows in the presence of a magnetic field and bear significance in the optimization of heat transfer under such conditions as suggested in [3].

FORMULATION & NUMERICAL SOLUTION

Rayleigh - Benard convection is first examined with the temperature gradient being parallel to the direction of gravity, in the presence of a uniform horizontal magnetic field, in the parameter range employed in [3]: aspect ratio A=1/20, Prandtl Pr=0.02, $c_H=4.15\times10^{-3}$, $c_S=4.5$. The latter two values correspond to the electric conductivity ratios between the wall material comprising the Hartmann (copper) and side (stainless steel) side walls, respectively and the liquid metal (NaK) flowing in the rectangular duct.
Three dimensional stability analysis is performed on a two dimensional base state, by allowing 3d disturbances in the spanwise direction of the duct. Continuity, 3d momentum, energy and electric charge conservation equations are solved for via a generalized eigenvalue problem that arises from the finite element methodology, $[J - sB]x = 0$, where J, B, x, s denote the Jacobian and mass matrices, the eigenvector and the corresponding eigenvalue, respectively. In order to allocate Hopf bifurcations we adopt a Cayley transformation that introduces complex constant σ. In this fashion we calculate the eigenvalues s that are close to parameter σ, via the Arnoldi method which calculates eigenvalues $1/(s-\sigma)$ with the highest absolute value. The most demanding part of the calculation is the Jacobian matrix inversion. Consequently, we opted for domain decomposition along the direction of the magnetic field, that facilitates parallelization of the inversion process via ScaLAPACK (Scalable Linear Algebra PACKage).
Next the interaction of two neighboring recirculating rolls, like those emerging as a result of the primary thermal instability in the above study, was investigated. These rolls are more or less aligned with the magnetic field, therefore its effect on their stability is marginal [4]. In the present analysis they arise as a result of a large volumetric heat production rate in a duct of square cross section [5] while the intensity of the magnetic field is maintained to a relatively low level. The above methodology is employed here as well in order to identify the dominant eigenvalues and corresponding eigenvectors. Furthermore, the GMRES (General Minimal Residual) method is applied and the Stokes

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operator is used as a preconditioner in order to perform continuation on the most important eigenvalues in the parameter space, Grashof or wave-number of the unstable mode, and construct the neutral stability diagrams with increased accuracy and minimal CPU requirements.

RESULTS

First the Rayleigh - Benard case is examined, in the presence of a magnetic field with increasing intensity. As Ha increases the vorticity component parallel to the magnetic field dominates the perpendicular component $(\omega_z^2+\omega_y^2)^{0.5}$, (Fig.1b,1c). As Ha increases the above numerical methodology captures $Gr_{Cr}\sim Ha^2$ with great accuracy; e.g when Ha=800 $Gr_{Cr}\approx 1030000$ when the experimental value from [3] is ~1070000 while quasi 2d analysis gives $Gr_{Cr}\approx 855000$. Furthermore, the nature of the eigenvector does not conform with the quasi 2d structure and the side and Hartmann layers are not well defined.

Stability analysis results, on the base state determined by the linear temperature profile, do not support the onset of travelling wave modes. Consequently the experimental finding of transient convection in [3] cannot be explained as a result of a thermal instability. Therefore, stability analysis is carried out on a 2d flow arrangement obtained when a constant volumetric heat production rate produces 2 counter rotating vortices, Fig.2a. In all cases examined, 3d disturbances are less stable than 2d ones, while increasing the Ha number increases critical Gr as well. Pertaining to low Pr and relatively high volumetric flow rate, the travelling wave mode dominates followed by a standing wave mode. Both modes are not associated with the modes dominating the 2d dynamics of the cavity and appear as a result of an elliptic instability, figs 2b,c, due to curvature of the stream lines in the base flow configuration. The critical Gr based on the heat production rate $PrSGr_{Cr}$ is equivalent to the Gr_{Cr} obtained for the onset of time dependent convection in [3].

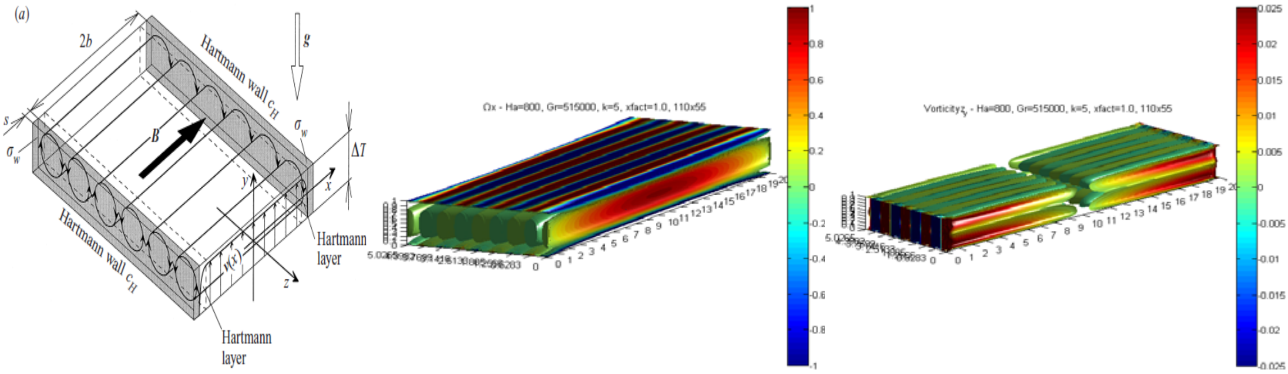

Fig. 1: (a) Geometry and flow pattern in [4], (b,c) x-component of vorticity and $(\omega_z^2+\omega_y^2)^{0.5}$, respectively; Ha=800, Gr_{Cr} =515000, A=1/20.

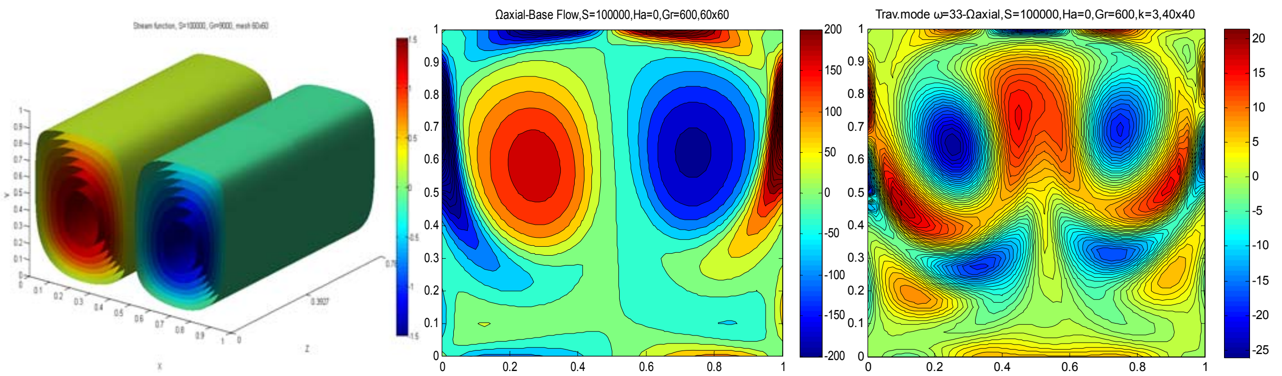

Fig. 2: (a) Base flow streamline pattern; and axial vorticity contours of the (b) base flow configuration, (c) dominant travelling wave disturbance.

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