MULTIPLE BUCKLING STATES OF FALLING VISCOUS THREADS

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Summary In addition to the well-known 'liquid rope coiling' instability, viscous threads falling onto surfaces can exhibit several other novel buckling states: periodic folding with simultaneous slow rotation of the folding plane; secondary periodic buckling ('supercoiling') of the cylindrical column produced by a primary coiling instability; and periodic column collapse. Laboratory experiments reveal a rich phase diagram of these states in the parameter space of the fluid viscosity, the flow rate and the fall height. Theoretical models based on the equations for thin viscous threads and shells provide quantitative understanding of the observed buckling frequencies and of some key aspects of the phase diagram.

LABORATORY OBSERVATIONS OF MULTIPLE BUCKLING STATES

One of the simplest experiments in fluid mechanics is to let a thin stream or thread of viscous fluid fall from a height onto a surface. The best-known phenomenon associated with this configuration is the 'liquid rope coiling' instability in which the falling thread winds itself into a whirling corkscrew-shaped structure just above the surface [1-4]. However, we have recently discovered that coiling is only one of five distinct states that a falling thread can exhibit (Figure 1). The simplest of the four additional states is axisymmetric stagnation flow (S), in which the fluid spreads radially over the surface. Next is a 'rotatory folding' state (F), in which the thread folds periodically to form a quasi-planar structure that simultaneously rotates about the vertical axis with a much lower frequency. The third is a novel 'supercoiling' (SC) state in which the cylindrical column formed by a first coiling instability undergoes a secondary periodic buckling instability with a much lower frequency. Finally, the tall cylindrical column generated by a rapidly coiling thread can undergo secondary buckling in the form of periodic collapse (PC).

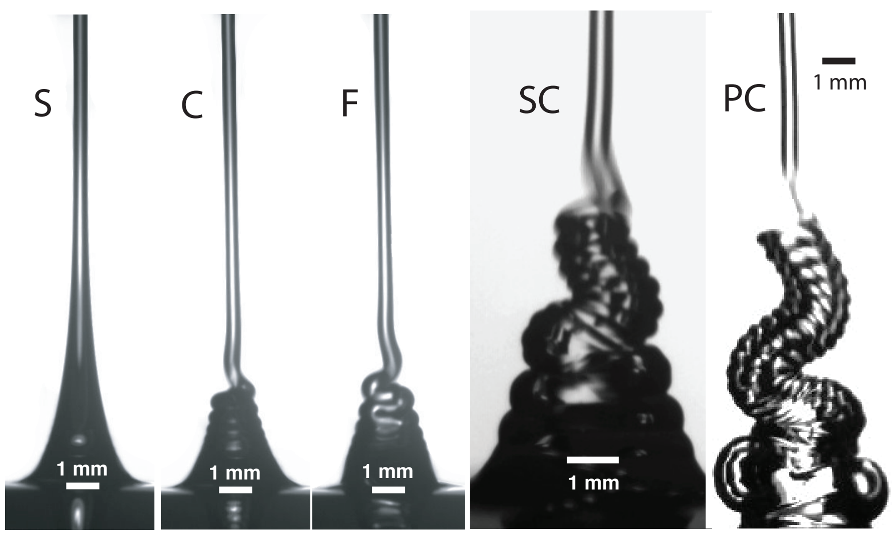

Figure 1. Five states of a viscous thread falling onto a surface: stagnation flow (S), steady coiling (C), periodic folding (F), supercoiling (SC), and periodic column collapse (PC).

Which state or combination of states is observed in the laboratory depends principally on three parameters: the fluid viscosity ν; the volumetric flow rate Q; and the fall height H. Figure 2 shows the rich phase diagram of these states as a function of Q and H for silicone oil with viscosity $\nu = 3350$ cSt. Roughly speaking, the states involving two buckling instabilities (PC, SC) are seen at low flow rates, those involving only one buckling instability (C, F) at intermediate flow rates, and the one without buckling (S) at high flow rates.

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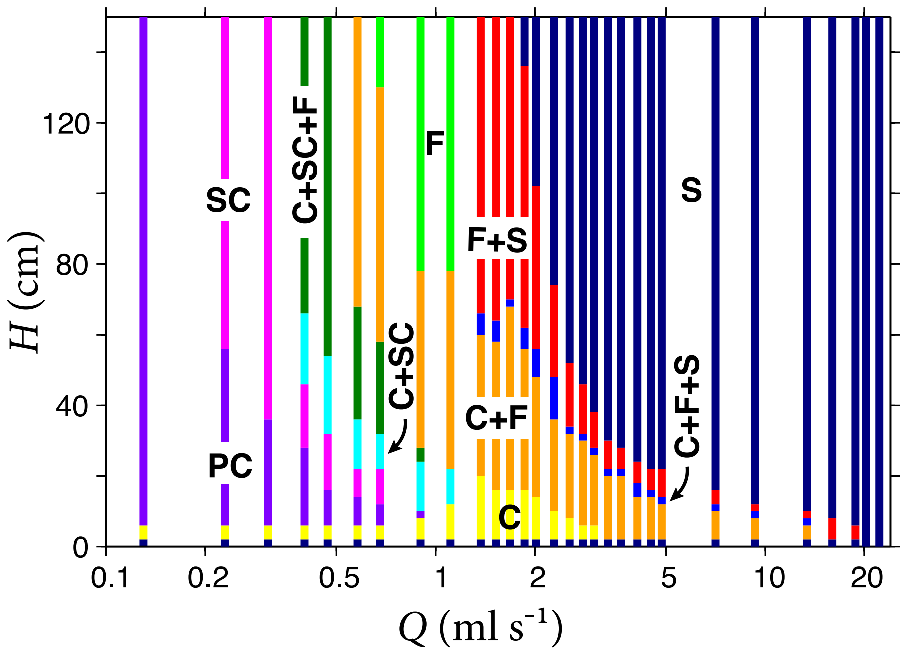

Figure 2. Phase diagram for the states shown in Figure 1 as a function of the flow rate Q and the fall height H, for silicone oil with viscosity $\nu = 3350$ cSt.

THEORETICAL UNDERSTANDING OF THE PHASE DIAGRAM: CURRENT STATUS

Of the four states in Figure 1 that involve buckling (C, F, SC, PC), coiling (C) is special because it is stationary when observed in the co-rotating reference frame. This makes it possible to obtain accurate numerical predictions of the thread's shape and coiling frequency by using a continuation method to solve the equations governing the steady-state behavior of a viscous thread [3]. Furthermore, by continuing these finite-amplitude solutions one can locate the critical surface in the ν-Q-H parameter space where the solutions cease to exist, and which should correspond to the boundary between coiling (C) and stagnation flow (S) in the phase diagram. This procedure reveals that the critical surface has three asymptotic limits corresponding to different balances among the viscous, gravitational and inertial forces that control coiling. The position of the critical surface agrees well with the C-S boundary in experiments with low-viscosity (300-2300 cSt) silicone oil [5].

WORK IN PROGRESS

Building on the above results, we are currently extending our numerical determination of the critical surface to higher viscosities to permit comparison with the phase diagram of Figure 2. A second goal is to understand the physical mechanisms responsible for the unique character of the states that involve secondary buckling (SC, PC), using experiments to quantify the frequencies of primary coiling and secondary buckling and a theoretical 'viscous shell' model to perform a stability analysis of the column produced by the primary coiling.

References

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