EXCITATION OF GÖRTLER-INSTABILITY MODES DUE TO SURFACE VIBRATIONS: THEORY AND EXPERIMENT

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Summary An excitation and growth of nonstationary Görtler vortices of different frequencies and spanwise wavenumbers in a Blasius-like boundary layer over a concave wall was studied experimentally only recently. In-depth analysis of the experimental data provided detailed databases on both the generation of the control flow disturbances due to surface vibrations and the growth characteristics of Görtler vortices in the boundary layer at relatively small Görtler numbers. Particularly, the shapes and amplitudes of vibrations of a specially designed flush mounted membrane were documented at different regimes of its excitation. The data are important to provide adequate boundary conditions for numerical modeling of the experiment. In the presentation, results of corresponding computations performed in the framework of linear approach are discussed. A novel algebraic-order reduction of governing discretized equations was used. It is shown that the nonparallelism of the basic flow streamlines (the downstream growth of the boundary layer) has a considerable effect on the receptivity and instability of Görtler vortices in the range of parameters under consideration. When the flow nonparallelity is taken into account properly, the experimental and numerical results are in a good agreement with each other.

INTRODUCTION

Using the technique of normal modes and assuming the local parallelism of basic flow streamlines H. Görtler showed [1], that stationary vortices, called later the Görtler vortices, could grow in boundary layers at concave walls. Their growth is possible when a special dimensionless parameter Gö (the Görtler number), whose value describes the ratio of centrifugal and viscous forces in the boundary layer exceeds a limit which depends on the spanwise vortex scale characterized usually by the spanwise wavenumber β or wavelength λ. The Görtler instability (both stationary and nonstationary vortices growing downstream) can really appear in shear flows near curved walls and is often responsible in experiments for amplification of streamwise counter-rotating vortices, whose growth is able to lead to premature laminar-turbulent transition [2].

However, it is well known that increments of the vortex amplitudes obtained in a wide range of β in different laboratory tests do not correspond to the Görtler's predictions. One of the main conjectures of this discrepancy is related to a nonparallelism of basic-flow streamlines due to the boundary layer growth [3] which cannot be adequately taken into account in the framework of normal modes at values of Gö of about 1 to 10 and λ of about and greater than the local boundary layer thickness. This leads to a formulation of initial-boundary-value problem for disturbances with governing equations parabolic in the streamwise direction. Consequently, the vortex development depends substantially on location and characteristics of their source.

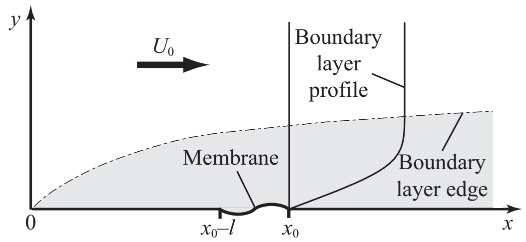

Figure 1. Flow layout in plane.

The excitation and growth of quasistationary and nonstationary Görtler vortices of different frequencies and spanwise wavenumbers in a Blasius-like boundary layer over a concave wall was studied experimentally only recently [4]. The experiments were carried out in a low-turbulence wind tunnel with the help of a specially designed disturbance source with a flush mounted membrane extended in the spanwise direction which makes it possible to generate streamwise-localized time-dependent surface inhomogeneities with a prescribed frequency f and the spanwise wavelength λ, see Fig. 1. Here x and y are the streamwise and wall-normal coordinates, respectively, $U_0 = 9.18$ m/s is the free-stream velocity, $l = 0.004$ or 0.0065 m is the membrane length and $x_0 = 0.27$ m is the coordinate of the membrane trailing edge (the surface curvature is 8.37 m). In-depth analysis of the experimental data provided detailed databases on both the generation of the control flow disturbances due to the surface vibrations and the growth characteristics of Görtler vortices in the boundary layer at relatively small values of Gö and different values of f and λ. Particularly, the so-called receptivity coefficients were estimated as well as the shapes and amplitudes of vibrations of the membrane were documented at different regimes

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of its excitation. The latter data are important to provide adequate boundary conditions for numerical modeling of the experiments.

PROBLEM FORMUATION AND RESULTS

At first, the experimental setup and results are discussed briefly in the presentation. Then, the governing equations are formulated. Essentially, they are the linearized Navier–Stokes and continuity equations for disturbances of Görtler-vortex type as can be found, e.g., in [5]. Special consideration is given to the corresponding numerical approach. It is developed based on some novel ideas including the algebraic-order reduction technique applied with a great success previously for numerical analysis of wall corrugation on temporal stability of plane flows [6]. Finally, results of the computations are presented.

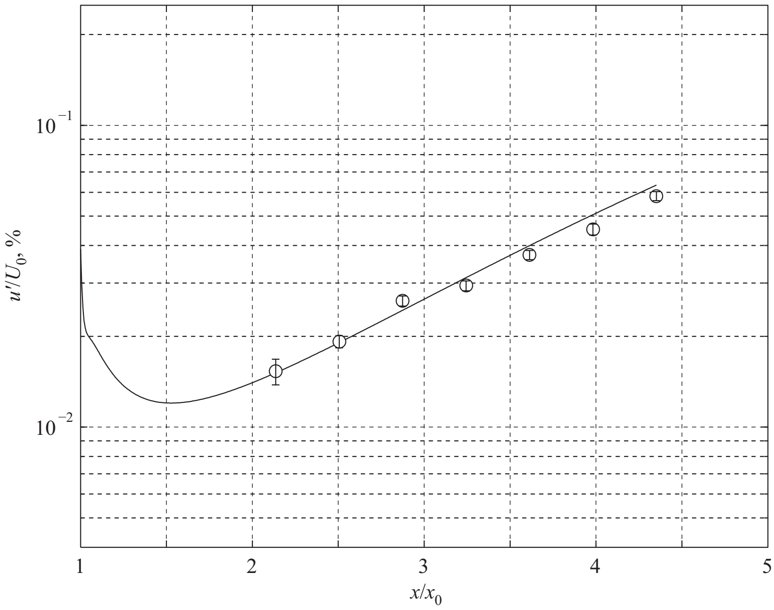

Figure 2. Disturbance growth: experimental (○) and numerical (—) data.

Some representative results are shown in Fig. 2. The experimental and numerical data on the growth of streamwise disturbances excited at $f = 11$ Hz and $\lambda = 0.008$ m are given in the figure. The amplitudes u' are taken at the disturbance maximum in the wall-normal direction. The numerical results are obtained on solving the linearized initial-boundary-value problem, which takes into account the characteristics of the membrane oscillations obtained in the experiments. As seen, the experimental and numerical results are in a good agreement with each other. Note, that the correspondence between the data is achieved only when the nonparallel terms are preserved in the governing equations. Hence, the nonparallelism of the basic flow streamlines has a considerable effect on receptivity and instability of the Görtler vortices in the range of parameters under consideration.

This has a strong influence on theoretical and practical aspects of definition and determination of the receptivity coefficients that are also discussed in detail. The knowledge of receptivity coefficients is important, e.g. in the variable N-factor technique or its modifications for predicting the laminar-turbulent transition in industrial applications [2]. In contrast to the Tollmien–Schlichting and crossflow instabilities, in the case of Görtler instability, it could be more appropriate but also more complicated in practice to determine the receptivity coefficients in respect to the optimal vortices (the vortices experiencing the maximum possible grows at a position of observation x) rather than to a sole leading instability mode.

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