AN EXPERIMENTAL STUDY ON THE RICHTMYER-MESHKOV INSTABILITY OF A MINIMUM-SURFACE INTERFACE WITH SINUSOIDAL BOUNDARIES

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Summary The interaction between a 3D interface and a planar shock wave has been experimentally studied in a shock tube. A controlled minimum-surface initial shape is given to the Air-to-SF_6 interface by fixing the soap film on two sinusoidal lines, which are mounted on the wall of the test-section. Sequences of the superimposed schlieren images reveal different behaviors of the instability growth near the wall and in the symmetry plane of the test section. In the former case, the instability growth agrees well with the predictions of the popular nonlinear theoretical models. While, in the latter case, measurements of the interfacial amplitude show a slower instability growth rate than that from the theoretical models as well as other 3D single-mode experimental results. It is a reasonable hypothesis that the instability growth of the shocked 3D interface is closely related to the principal curvatures of the interface.

INTRODUCTION

The Richtmyer-Meshkov instability (RMI) is a fundamental hydrodynamic instability that occurs when an impulsive acceleration is exerted on the perturbed interface between two different fluids. During the passage of the shock wave through the interface, the baroclinic vorticity deposits on the interface and drives the flow from a short stage of linear perturbation growth [1] to a stage of nonlinear instability growth and transitions to turbulence eventually. RMI has a significant meaning in a variety of scientific fields, such as inertial confinement fusion (ICF) and supersonic combustion.

Realistically, a three-dimensional (3D) interface commonly appears in the shock-inhomogeneity interactions. There is a scarcity of experimental results for the RMI model of a 3D interface with the exact initial shape [2, 3]. Here we describe the RMI of a well-controlled minimum-surface interface whose boundary is the sinusoidal line, the amplitude relative to its wavelength is $k\eta_0 \approx 0.90$. Our purpose is twofold, the first is to experimentally study the RMI of a shocked 3D interface with the controlled initial shape, the other is to study the instability growth of the perturbation with a medium-sized initial amplitude.

EXPERIMENTAL METHODS

The data presented here are acquired in a horizontal shock tube with an open-end test section whose cross section is 140×20 mm^2. A planar shock wave is released into the driven section (2.5 m) by puncturing the diaphragm just before the driver section (1.7 m). The evolutions of the flow are captured by a high-speed video camera (Photron FASTCAM SA5) in a schlieren system. Initial conditions are produced by fixing the soap film on the sinusoidal lines on the up and down wall of the test section with 2h = 20 mm (Figure 1(a)). The initial pressure across the interface between air and sulfur hexafluoride (SF_6) is the same so that the interface can be characterized as a minimum surface [4]. As shown in Figure 1(a), the sinusoidal line is described by the function: $\eta(y, 0) = \eta_0^- \sin(ky), y \in [0, 140]$ mm, where $k = 2\pi/\lambda$, the initial amplitude is η_0^- = 5 mm, the wavelength is λ = 35 mm, and the superscript "-" denotes parameters before the impaction of the shock wave.

RESULTS AND DISCUSSION

The interface is three dimensional as described in Figure 1(a). Positions of the tips of the interface at z = 0 and z = $\pm h$ are denoted as IL, IR and OL, OR respectively. At z = 0, the interfacial contour is approximated to a sinusoidal perturbation with the initial amplitude η_1^-. We assume the interface near the the tip of the interface can be approximated to a catenary: $x = -\cosh(k^2\eta_1^- z)/k^2\eta_1^-$. This is a theoretical solution for the minimum surface with two parallel circular boundaries. The amplitude η_1^- can be determined as: $\frac{1+k^2\eta_1^-\eta_0^- - k^2\eta_1^{-2}}{\cosh(k^2\eta_1^- h)} = 1$. In the present work, $\eta_1^- = 1.88$ mm. Figure 1(b) shows the motion of the shocked minimum-surface shaped interface. The contour of the shocked interface quickly distorts as the baroclinic vorticity deposits on the interface. Owing to the integration effects in the schlieren system, the interfacial contour at z = 0 and z = $\pm h$ can be observed at the same time. It is clearly observed that the instability at z = $\pm h$ grows faster than that at z = 0.

For a planar air/SF_6 interface, impacted by a planar shock wave, the velocity jump of the interface $\triangle v$ can be calculated from one-dimensional (1D) gas-dynamics. For a concave interface to the front of the shock wave, the slightly converged reflected shock and diverged transmitted shock near the tip of the interface result in a pressure perturbation across the interface, which induces an increment of the shock induced velocity compared to the 1D case. For a shocked convex interface, the velocity is smaller than the 1D case. In the present experimental results, the incident shock Mach number is M_S = 1.17, the velocity jump is $\triangle v$ = 60.82 m/s from 1D gas-dynamics. The velocities of OL, OR, which have a larger curvature at z = $\pm h$, are 50.03 and 65.45 m/s, respectively, while the velocities of IL, IR at z = 0 are 57.06

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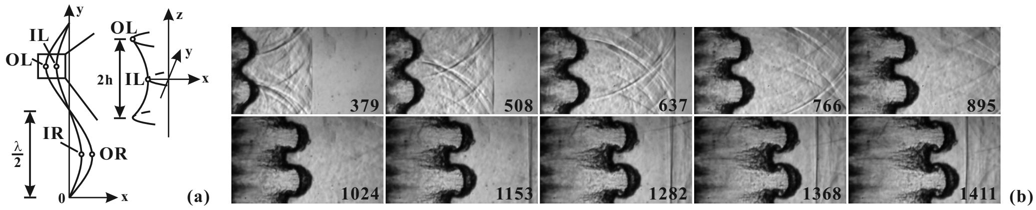

Figure 1. (a) Sketch for the minimum surface with sinusoidal boundaries, and the 3D view of the marked region. IL, IR and OL, OR denote the position of the left and right tips of the interfacial contour at z = 0 and z = ±h. (b) Series of the superimposed schlieren images taken from the single Air-to-SF_6 experiment with M=1.17. Numbers indicate time after the shock passes x = 0 in μs.

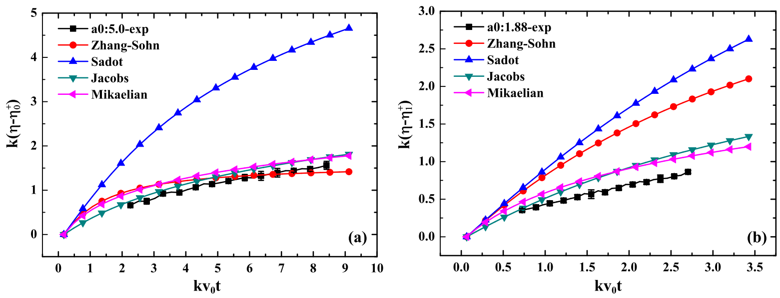

Figure 2. Dimensionless evolution of the amplitude at z = $\pm h$ (a) and z = 0 (b), where k is the wavenumber, η is the amplitude of the evolving perturbation, and the superscript "+" denotes parameters after the shock wave. Error bars represent the RMS error of the measurements.

and 62.43 m/s. It is a reasonable hypothesis that a larger curvature corresponds to a larger derivation from the 1D gas-dynamics result. The pressure perturbation, in the case of the 3D interface, is related to the both principle curvatures near the tips of the interface and then results in the instability growth.

The perturbation amplitude η is defined as a half of the mixing width between OL and OR (z = $\pm h$) and IL and IR (z = 0). Figure 2 shows the dimensionless instability growth of the experiment, compared with the popular 2D nonlinear theoretical models [5, 6, 7, 8]. It can be seen that the large-amplitude effect is in accordance with others' experimental results. The interfacial contours, near the wall of the test section at z = $\pm h$, are dominated by the sinusoidal boundaries, and the growth of the perturbation amplitude agrees well with the predictions of several nonlinear theoretical models (Figure 2(a)). In the symmetry plane of the test section at z = 0, two principal curvatures are at the corresponding magnitude and opposite in direction relative to the propagation of the shock, therefore, the instability growth rate is smaller than the predictions of the 2D theoretical models (Figure 2(b)).

CONCLUSIONS

The RMI of the shocked 3D minimum-surface shaped interface, with a sinusoidal boundary ($k\eta_0 \approx 0.90$), is experimentally studied using the high-speed schlieren method. The curvature of the interface relative to the front of the shock wave results in a derivation of the velocity jump of the shocked interface compared with 1D gas-dynamics. As the passage of the shock wave through a minimum-surface interface, whose principle curvatures are at the opposite direction, the instability growth becomes slower than the results of 3D single-mode RMI [2, 3], as well as several nonlinear models which are validated by the 2D instability growth.

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