THE RELIABILITY OF THE IMPROVED E-N METHOD FOR BOUNDARY LAYER TRANSITION PREDICTION ON A FLAT PLATE

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Summary The transition criterion in the improved e-N method is that transition would occur whenever the velocity amplitude of disturbance reaches 1-2% of the free stream velocity, instead of the empirical N factor. In this paper the reliability of this key assumption in the improved e-N method is checked by solving the Parabolized Stability Equations (PSE). Transition prediction for an incompressible boundary layer and a hypersonic boundary layer on a flat plate is investigated. The results show that transition locations predicted by the two methods agree reasonably well with each other, which provides direct evidence for the transition criterion proposed in the improved e-N method.

INTRODUCTION

The prediction of laminar-turbulent transition in boundary layer has always been an important problem in fluid mechanics both from theoretical and practical points of view. Yet the most popular method for its prediction, the so called e-N method, is largely a semi-empirical method[1]. Su & Zhou[2]-[3] have analyzed the problems existing in the conventional e-N method and proposed certain improvements, so to reduce its dependence on experiments or experience. The improvements include a new transition criterion and some considerations of receptivity.

Obviously, to make the prediction method reliable and more rational, one has to check, first, how big the error would be in using the parallel assumption; and second, what would be the error in using the new transition criterion, i.e. transition would take place whenever the velocity amplitude of disturbance in the improved e-N method reaches 1-2% of the free stream velocity.

In fact, the first problem mentioned above is not a serious one, because the results from linear stability theory (LST) have already been compared many times with those from direct numerical simulation and results from applying the parabolized stability equation (PSE). The conclusion was, when the Reynolds number is large, which is exactly the case for most transition problem, the difference was insignificant. So our main concern would be the second problem, i.e. if the new transition criterion is reasonable.

We would solved this problem by comparing results from applying e-N method and by using PSE, as it has been shown in [4] that results from applying PSE in predicting the transition location is comparable with those from DNS, provided the initial conditions are the same in both methods.

VERIFICATION OF THE IMPROVED E-N METHOD

Initialization of PSE Calculations

In the e-N method, one seeks to find the T-S wave, whose amplitude reaches the given threshold for transition first among all possible T-S waves. While in PSE, a single T-S wave cannot trigger transition, one has to choose more than one T-S waves, in order to finally trigger transition. So the problem is how to choose the initial set of T-S waves, which is comparable with the single wave in e-N method.In nonlinear stability theory, there are famous models for nonlinear instability, i.e. the resonant triad and secondary instability. The initial disturbances are modeled by a 2-D wave and a pair of oblique waves with the same frequency, half of the 2-D wave, but opposite spanwise wave numbers, because the interaction between such waves would be strongest. So we also take similar model for our PSE method, i.e. the initial disturbances consist of a 2-D T-S wave and a pair of oblique T-S waves having frequency the half of the 2-D wave.

It seems there can be infinite set of oblique waves, as it can have arbitrary span-wise wave number. However, in the improved e-N method, only waves with a real group velocity are considered, here the same principle is adopted. Then the span-wise wave number of the oblique waves can be fixed. In the improved e-N method, it is assumed that all waves have the same initial amplitude 0.3‰ of the free stream velocity. In the PSE method for comparison, the same assumption should be used, so the amplitude of either of the two oblique waves should be 0.15‰.

The computation of PSE method ends until the wall shear stress rises abruptly, which is assumed to be the location of transition.

An incompressible case

The transition of an incompressible boundary layer is investigated. The Reynolds number is 2000, based on the displacement thickness of the boundary layer at the inlet of the computational domain, the velocity, density and viscosity coefficient of the free-stream. The basic flow is given by the similarity solution.

At first the improved e-N method is used for predicting the transition location. It is done as follows: the computation

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starts from the location where the amplitude of the disturbance wave can be reasonably estimated. The transition location is so determined that among all the locations that a certain T-S wave's amplitude reaches the threshold amplitude A_{tr} for transition, the most upstream one is the location of transition. The integration starts from the inlet of the computational domain.

Figure 1 shows the results of both methods. The most upstream point on each curve determines the transition location under the respective criteria of transition. For example, for the improved e-N method, the predicted transition location would be x=360, 450, 500, corresponding to A_{tr}=0.01, 0.02, 0.03 respectively, while for the PSE method, the transition location predicted is x=440. In fact, if in the improved e-N method, Atr is taken to be 0.018, which is within the range 1-2% of free stream velocity as mentioned in the above introduction, then the transition location predicted by both methods would be the same. By the way, the frequencies of the 2-D waves triggering the transition in both methods are also roughly close to each other.

A hypersonic case

The transition of a hypersonic boundary layer is also investigated in the similar way. The Mach number of the oncoming flow is M=6. The Reynolds number is 36000, defined in the same way as the incompressible case above. The wall temperature condition is adiabatic. The initial condition of disturbances is the same as the above case.

Figure 2 shows the comparison of the results obtained by both methods. The transition location obtained by the PSE method is *x*=80, which is exactly in the range of transition location predicted by the improved e-N method using the transition criterion A_{tr}=0.01, 0.02, respectively. If in the improved e^N method, A_{tr} is taken to be 0.013, the transition location predicted by both methods would be the same. The transition criterion adopted in the improved e-N method is reasonable. Considering both the incompressbile and hypersonic cases the transition criteria in the improved e-N could be taken as A_{tr}=0.015. The frequency of the wave triggering the transition is also slightly bigger than that in the improved e-N method, similar to the incompressible case. The reason is thought to be the influence of the nonparallelism and nonlinear effects. More work is underway for further verification.

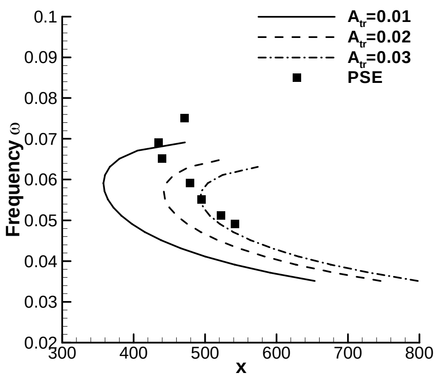

Fig. 1 Transition location for incompressible flow

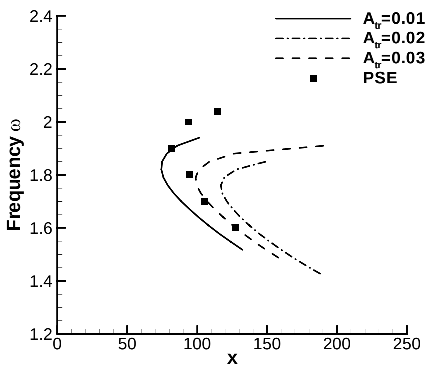

Fig.2 Transition location for hypersonic flow

CONCLUSIONS

The conventional e-N method is a semi-empirical method, with no consideration on receptivity, and no consideration on physical criterion for transition. In the improved e-N method, the initial location of integration and the initial amplitude of the T-S wave are determined by a certain consideration on receptivity mechanism, and the transition criterion is taken to be that the disturbance wave's amplitude reaches 1-2% of the free stream velocity, as indicated from results of DNS for transition. However, it still bears some simplifications, which should be checked by other more sophisticated method. In this paper, PSE method is used to check the results from the improved e-N method. The result of comparison does provide evidence that the improved e-N method bears some rationality.

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