SECONDARY STABILITY IN SHEAR THINNING PIPE FLOW

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Summary Secondary linear stability analysis of pipe flow of shear-thinning fluids modulated azimuthally by finite amplitude streaks is performed. The streaky base flows considered are obtained by extracting the velocity fields from two-dimensional direct numerical simulations using finite amplitude longitudinal rolls as initial condition. The sign of the temporal growth rate determines the stability condition. It is found that the onset of instability is delayed with increasing shear-thinning effects.

INTRODUCTION

We consider the flow of an incompressible purely viscous shear-thinning fluid in a circular pipe of radius $\hat{R}$. The governing equations in dimensionless form are :

$$\boldsymbol{\nabla} \cdot \boldsymbol{U} = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{U}}{\partial t} + (\boldsymbol{U} \cdot \boldsymbol{\nabla})\, \boldsymbol{U} = -\boldsymbol{\nabla} P + \boldsymbol{\nabla} \cdot \boldsymbol{\tau}, \tag{2}$$

where P is the pressure, including the gravity effect, and $\boldsymbol{\tau}$ the deviatoric stress tensor. For purely viscous fluid, *i.e.* fluids for which the viscosity depends only on the shear rate, the deviatoric shear stress tensor is $\boldsymbol{\tau} = \frac{1}{Re}\mu \dot{\boldsymbol{\gamma}}$ with $\dot{\boldsymbol{\gamma}} = \boldsymbol{\nabla} \boldsymbol{U} + (\boldsymbol{\nabla} \boldsymbol{U})^T$, the strain-rate tensor. The Reynolds number is defined with the viscosity evaluated using the shear-rate at the wall, $Re_w = \frac{\hat{\rho}\hat{W}_0\hat{R}}{\hat{\mu}_w}$.

The dependence of the viscosity on the shear rate is described b the Carreau model: $\mu = \mu_\infty + (1-\mu_\infty)\left[1 + (\lambda\dot{\gamma})^2\right]^{(n_c-1)/2}$. Rheological parameters are fixed to $\lambda = 30$, $n_c = 0.5$ and $\mu_\infty = 0.001$, which are in the range of experimental data. One can note that the Newtonian fluid is recovered by setting any of the limits: $n_c = 1$, $\lambda = 0$ or $\mu_\infty = 1$. The dimensionless second invariant of the strain rate tensor is $\dot{\gamma} = \left[\frac{1}{2}\dot{\gamma}_{ij}\dot{\gamma}_{ij}\right]^{1/2}$.

BASE FLOW: STREAK GENERATION

In the framework of the linear stability theory of the 1D laminar flow of Carreau fluid in a cylindrical pipe, it is shown, for a large range of the rheological parameters, that the perturbation which leads to the maximum amplification of the kinetic energy consists in a pair of longitudinal counter-rotating rolls [1], streamwise vortices of amplitude ϵ_0 (ratio of the energy norm of the disturbance to that of the laminar flow) generate streaks of high and low streamwise velocity with an amplitude $O(\epsilon_0 Re)$ over a time $O(Re)$. For sufficiently strong intensity of the steamwise vortices, and therefore for large transient growth of the streaks, the nonlinear terms come into play. The axial velocity profile is flattened via the Reynolds stresses, which leads to a saturation of the energy transient growth of the streaks. The nonlinear evolution of the flow structure under a constant pressure gradient is computed in 2D situation using a solenoidal pseudo-spectral Petrov-Galerkin method similar to that proposed by [II]. Fourier series are used in the azimuthal direction and Chebychev-like polynomials in the radial direction. The computations were performed for $1000 \leq Re_w \leq 3000$. Streamwise velocity contour plots in the cross-section (r, θ) are shown in Fig. 1 for Newtonian and Carreau fluids and two particular values of Re_w at $t = 50$.

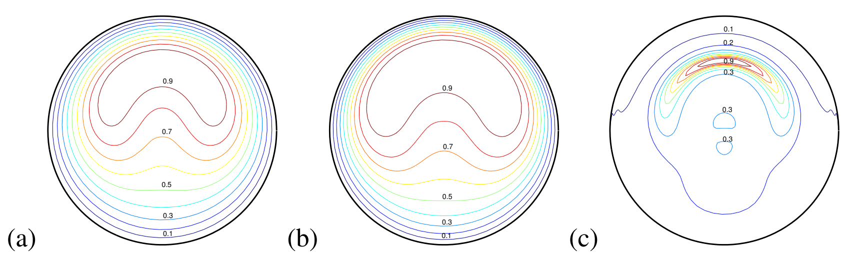

Figure 1. Nonlinear base flows. Normalized contours of constant streamwise velocity $\tilde{W}_b(r,\theta)/\tilde{W}_{b,max}$ in (r,θ) section extracted from the direct numerical solution at $t = 50$. **(a)** Newtonian fluid at $Re = 1500$, $W_{b,max} = 0.9028$. **(b)** Carreau-fluid at $Re_w = 2500$, $W_{b,max} = 0.9559$. **(c)** Contours of constant viscosity μ for the Carreau fluid.

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RESULTS AND DISCUSSION

Solutions to the linearized perturbation equations around the 2D streaky flow are sought in the form of normal modes $\boldsymbol{u}(r,\theta,z,t) = \exp\left(ikz + \sigma\, t\right) \sum_{n=-\infty}^{+\infty} \boldsymbol{u}_n(r) \exp(in\theta)$. As a result, a large coupled system of algebraic equations of the form $\boldsymbol{A}\boldsymbol{a} = \sigma \boldsymbol{B}\boldsymbol{a}$ is obtained for a given axial wavenumber k. The generalized eigenvalue problem is solved using QZ-algorithm implemented in Matlab package. The eigenvalues are considered converged if with increasing truncation (M,N), for instance from $(16,16)$ to $(22,22)$, the relative variation is less than 0.5% for the first ten least stable eigenmodes. Figure 2 shows the temporal growth rate of the disturbance against the streamwise wavenumber k at different Reynolds numbers for Newtonian and Carreau fluids. The amplitude of the initial streamwise vortices is $\epsilon_0 = 10^{-3}$. For this particular value of ϵ_0, the critical conditions for the onset of linear instability are $k = 3.6, Re \approx 1450$ for Newtonian fluid and $k = 2.1, Re_w \approx 2450$ for Carreau fluid. It is found that the size of the initial perturbation $\sqrt{\epsilon_0}$ necessary to trigger instability varies with $Re^{-3/2}$ for both Newtonian and shear thinning cases (Fig. 3). Results are in agreement with previous numerical [2] and experimental [3] studies. One has to note a delay in the onset of instability for the shear-thinning fluid.

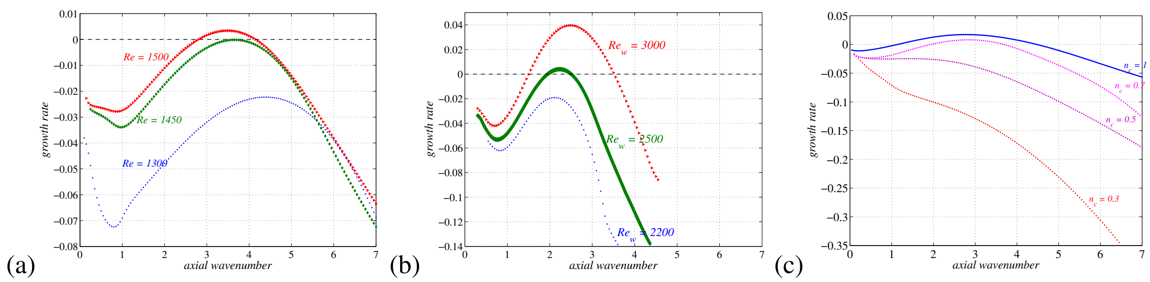

Figure 2. Dispersion curves for **(a)** Newtonian and **(b)** Carreau fluid ($n_c = 0.5$, $\lambda = 30$) at different Re_w. **(c)** Dispersion curves for the least stable mode at $Re_w = 2000$ for different shear thinning indexes n_c. In all cases, base flows are obtained from a non linear simulation with $\epsilon_0 = 0.001$ and $t = 50$.

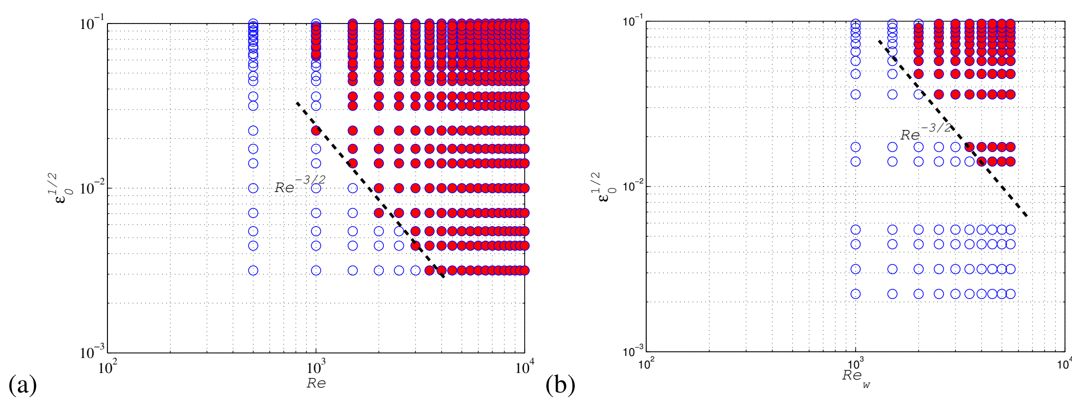

Figure 3. Stability regions for **(a)** Newtonian and **(c)** Carreau fluid ($n_c = 0.5$, $\lambda = 30$) at different Re_w and ϵ_0. Red dots represent those situations were the flow is linearly unstable, whereas white dots represent negative growth rate for all k.

CONCLUSION

Secondary stability analysis of pipe flow for Newtonian and shear-thinning fluids has been performed. The 2D base flow is obtained using 2D numerical simulation of the momentum equations, with finite amplitude ϵ_0 of the initial streamwise vortices. This secondary stability approach is less expensive than a fully non linear DNS and captures the main characteristics of the streak breakdown instability mechanism. Finally, the computations show that the onset of the linear instability depends on the size of the initial perturbation and it is delayed with increasing the shear-thinning effects.

References

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