HIGHLY DISSIPATIVE PERTURBATIONS IN A COUETTE FLOW

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Summary By means of a global variational procedure we identify non-linear optimal disturbances, yielding the largest energy growth at a given target time, inducing bursting events in a Couette flow. These disturbances are characterized by a basic structure, composed by inclined streamwise vortices along localized regions of low and high momentum, closely recalling the one found by non-linear optimizations in the boundary-layer flow. This indicates that this structure may be the most "energetic" one at short times for shear flows. Such perturbations pass close to the edge state, but they are rapidly repelled away from it, leading the flow to high values of the dissipation rate. After this dissipation peak, the trajectories spend some time in the vicinity of an unstable periodic orbit (UPO), leading us to conjecture that bursting events can be linked to heteroclinic orbits between the equilibrium solution on the edge and the UPO.

INTRODUCTION

Recently, many authors (see [1, 2]) have assumed that transition in shear flows is linked to the existence of exact coherent states (ECS), which are invariant solutions of the Navier-Stokes (NS) equations, containing the basic flow structures capable of inducing transition. Very recently, some authors [3] computed the initial condition on the laminar-turbulent boundary which represents the closest state to the laminar solution for a plane Couette flow. Their approach optimizes the route which goes directly to the statistically steady turbulent state, but, being based on time-averaged quantities, it cannot capture strong transient behaviours of the flow such as the bursting events, which are an important feature of the intermittent nature of turbulence. Therefore, in this paper we compute optimal perturbations which are able to induce high-energy transients in the flow at a short time. To this end, we use a Lagrange multiplier optimization in a non-linear framework, looking for the perturbations of the laminar base flow which are able to induce the largest perturbation energy growth at a short target time [4]. The mechanisms leading such non-linear optimal perturbations to bursting events and the role of the interactions with the ECSs will be analyzed in detail.

PROBLEM FORMULATION

The non-linear behaviour of a perturbation $\mathbf{q} = (u', v', w', p')^T$ evolving in the Couette flow is studied by employing the NS incompressible equations written in a perturbative formulation, with respect to the exact Couette solution. Dimensionless variables are defined with respect to half of the velocity difference between the two walls, U_w and half of the distance between the plates, h, so that the Reynolds number is $Re = U_w h/\nu$, ν being the kinematic viscosity. The aim is to find the initial perturbation $\mathbf{q}_0$ of given initial energy $E(0) = E_0$ which can induce at target time T the largest energy $E(T) = \langle \mathbf{u}(T) \cdot \mathbf{u}(T) \rangle$, where the symbol $\langle\rangle$ indicates integration over the considered three-dimensional domain. The objective function of the procedure is the energy gain $E(T)/E(0)$, and the three-dimensional NS equations and the value of the initial energy are imposed as constraints. The Lagrange multiplier technique consists in searching for extrema of an augmented functional, with respect to every independent variable. The gradient of the augmented functional with respect to the initial state $\mathbf{q}_0$ is forced to vanish by means of a conjugate gradient algorithm.

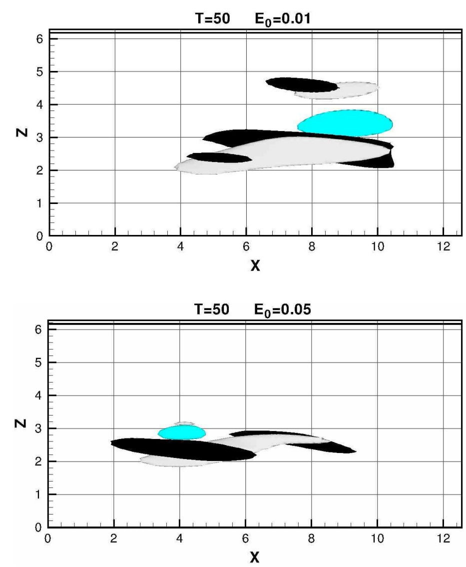

Figure 1. Iso-surfaces of streamwise velocity (blue), and vorticity (black and white) for the initial non-linear optimal perturbations for $T = 50$, $E_0 = 0.01$, and $E_0 = 0.05$ (from top to bottom).

RESULTS

The non-linear optimization procedure has been applied to the Couette flow with $Re = 400$. For values of E_0 and T large enough, the non-linear optimal disturbances are characterized by a basic structure, composed by inclined streamwise vortices on the flanks of localized regions of low and high momentum, as shown in Figure 1 for $T = 50$ and two values of the initial energy. Such a basic structure closely recalls the one found in other shear flows [4], indicating that this structure may be the most "energetic" one at short times. Despite the structure remains the same, when the values of E_0 and T are increased, some small modifications of the spanwise localization of the vortices are recovered, which can be observed by comparing the two frames of Figure 1. This induces to speculate about two different paths to transition which are studied in detail by means of direct numerical simulations (DNS). In particular, DNSs have shown that optimal disturbances obtained for large E_0 and T induce bursting events at short time, whereas for lower values of these parameters the flow is directly attracted towards the turbulent state. For this reason, the optimal disturbances have been classified in two classes, the *highly dissipative* and the *short-path* optimal disturbances.

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Both classes lead the flow to turbulence skipping the phases of streaks formation and secondary instabilities typical of the Waleffe regeneration cycle [2]. In particular, the non-linear transition scenario relies on the following steps: i) initial amplification of the streamwise component of velocity due to the Orr and lift-up mechanisms; ii) growth of the streamwise vorticity due to the coupling with the streamwise shear induced by the finite amplitude streamwise component of the perturbation; iii) sustainment and tilting of the vortices by feedback of the non-linear terms, inducing modulated streaks; iv) establishment of modulated streaks and inclined vortices which sustain turbulence. This transition scenario exploits three main features of the non-linear optimal disturbances in shear flows: i) the large initial values of the streamwise velocity component; ii) the streamwise dependence of the disturbance; iii) the presence of initial (inclined) streamwise vortices.

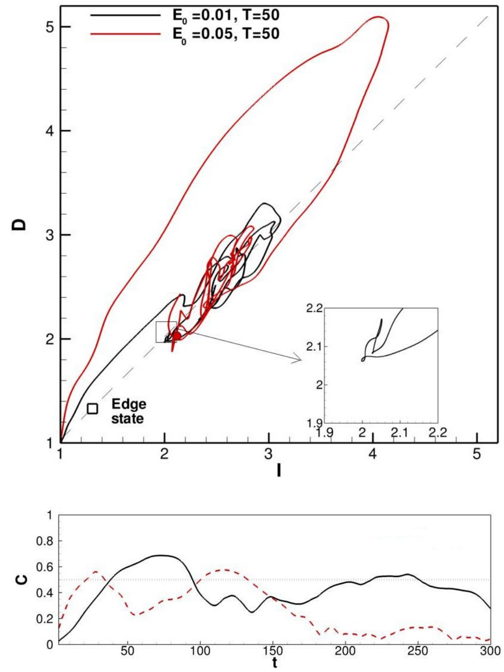

Figure 2. Trajectory on the $I - D$ phase plane (top frame) and correlation with the edge state (bottom frame) of a short path disturbance (black lines) and a highly dissipative one (red lines).

Although the mechanisms leading to transition are the same, the two classes of perturbations are found to behave very differently when looking at their trajectory on the $I - D$ (energy input - dissipation rate) projection of the phase space, which is provided in Figure 2. In particular, the short-path perturbations (black line) are found to spend a considerable amount of time in the vicinity of the edge state (which coincides with the equilibrium solution in [1]) in the first phase of their route to transition, as shown in the bottom frame of Figure 2, providing the correlation of the flow state in time with such an equilibrium solution. Also the highly dissipative optimal disturbances (red line) pass close to the edge state, but they are found to be rapidly repelled away from it, leading the flow to high values of the dissipation rate ($D \approx 5$). After this dissipation peak, the trajectories does not lead towards the turbulent attractor, but they spend some time in the vicinity of the unstable periodic orbit found in [5] (UPO). This links our results to the recent findings in [6], where it is conjectured that bursting events are due to passages along homoclinic orbits approaching the UPO. Along these orbits streamwise vortices are observed on the crests and valleys of the streaks, having significant dissipation close to the walls, which are usually not recovered in the regeneration cycle. These structures are very similar to the ones observed here when the laminar solution is perturbed by the highly dissipative perturbations, which are shown in Figure 3 for $t = 140$. Thus, we conjecture that a bursting event can be obtained also as an heteroclinic orbit between the equilibrium solution on the edge and the UPO. The highly dissipative perturbations are able to rapidly join such a heteroclinic orbit starting from the laminar solution, whereas the short path ones are strongly attracted towards the edge state, directly reaching the turbulent state. This different behaviour appears to be due to the different localization of the disturbances, inducing stronger local peaks of the dissipation rate in the former with respect to the latter case.

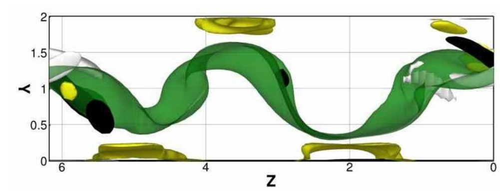

Figure 3. Null streamwise velocity contours (green), streamwise vorticity perturbation (black and white) and local dissipation rate (yellow) extracted at $t = 140$ from a DNS initialized by an highly dissipative optimal perturbation.

CONCLUSIONS

The results presented here point out a new rapid path to transition in a Couette flow, mostly relying on non-linear mechanisms, which reproduces the typical features of subcritical transition to turbulence in shear flows, such as the spatial and temporal intermittency characterizing the bursting phenomenon.

References

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