# THE STUDY OF VARIABLE SPECIFIC HEAT ON THE STABILITY OF HYPERSONIC BOUNDARY LAYER

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*Summary*: The previous studies of analyzing Blasius similar solution as a basic flow indicate that variable specific heat has a lot effects on the flow stability. In this paper, the basic flow is obtained by solving the control equations considering variable specific heat by using DNS. The results show that the basic flow indeed satisfies similar and very closes to the Blasius solution. In addition, flow stability analysis is also conducted on the basic flow by DNS and compared with that of constant specific heat (perfect gas).

## Introduction

It is difficult to obtain the accurate value of aerodynamic heating, which is the current difficulties in the study of hypersonic flow, for the great changes of the thermodynamic properties of gas within hypersonic flow. It is essential to determine the thermodynamic properties of gas. Last century, Tannehill and Mugge[1], Srinivasna [2], Gupta et al. [3] had a study in the thermodynamic properties of equilibrium air. In 2003, Iannelli [4] studied the thermodynamic properties of the chemically reacting air containing five species. The results of Jia and Cao [5] indicated that how specific heat varies with temperature must be considered in the study of boundary layer stability of hypersonic flow.

In this paper, the effects of specific heat as a function of temperature on the stability of flat plat boundary layer with Mach number 6 and 8 basing on linear stability theory are investigated, and compared with the perfect gas.

## Numerical method

The two-dimensional N-S equation [6] based on a rectangular coordinate system is as follows:

$$\frac{\partial U}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}=\frac{\partial E_v}{\partial x}+\frac{\partial F_v}{\partial y} \qquad (1)$$

The flux splitting with constant specific heat is list on the literature [6]. For the case of variable specific heat, the internal energy expression varies from $e=c_v T = RT/(\gamma\text{-}1)=p/\rho/(\gamma\text{-}1)$ of constant specific heat to $de=c_v(T)dT$.

The method in the literature [6] can not applicable in this case. The method according to literature [7] is used to split the flux in this paper. Taking the x-direction for example, the final positive and negative flux is as follows:

$$E^{\pm}=\left(l_1^{\pm} \quad ul_1^{\pm}+cl_2^{\pm} \quad vl_1^{\pm} \quad \frac{u_k^2}{2}l_1^{\pm}+ucl_2^{\pm}+\frac{c^2}{\gamma-1}l_3^{\pm}-\frac{\gamma_2}{\gamma_1}l_1^{\pm}\right)^T \qquad (2)$$

Compared with the case of perfect air, only $-\frac{\gamma_2}{\gamma_1}l_1^{\pm}$ is a more in the expressions of the flux with regard to the internal energy. Where $\gamma_1=\left(\frac{\partial p}{\partial(\rho e)}\right)_{\rho}$; $\gamma_2=\left(\frac{\partial p}{\partial \rho}\right)_{\rho e}$, the expressions of $l_i^{\pm}$ list in the literature [6]. $\gamma$, c are corresponding local specific heat ratio and the speed of sound.

After Steger-Warming splitting, the governing equations are solved by using the $5^{th}$-order upwind compact difference scheme for the nonlinear term, and using the $6^{th}$-order center compact difference scheme for the viscous term, and using $2^{nd}$-order total-variation-diminishing (TVD) Runge-Kutta scheme [8] for time integration.

When solving the equations, no-slip and adiabatic/isothermal wall conditions are adopted, and NSCBC suggested by Lele [9] on the normal upper boundary, derivative extrapolation for the outlet boundary condition, and Blasius solution for the inlet boundary.

In this paper, Mach number 6 and 8 are selected cases, and the oncoming flow gas parameters are taken as the air corresponding to the height 40000m.Re=10000. The computational domain is taken as $x_N \times y_N = 600\times 20$, and a grid of $600\times 200$ stretches in every direction.

### Verification of the program

To verify the program, the self-similarity results are obtained by calculating the longer flow to steady firstly; then y is dimensionless by using local displacement thickness, and compared with the Blasius solution. Here, the Blasius solution is also obtained under the assumption of variable specific heat, the specific heat as a function of temperature [5].

Fig.1 shows the comparison results of the different profiles of the basic flow to the Blasius solution and the $2^{nd}$ mode neutral curves of different profiles with Mach number 6.

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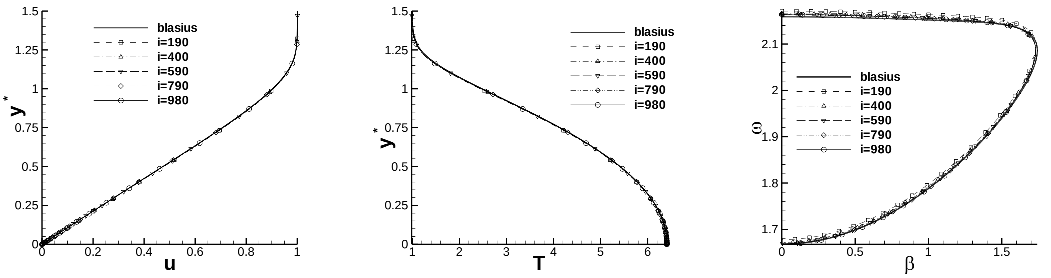
It can be seen from fig.1 that the different profiles of basic flow and the neutral curves of different profiles are good agreement with that of Blasius solution, which verify the correctness of the program of variable specific heat, and the existence of similarity solution under the condition of variable specific heat.



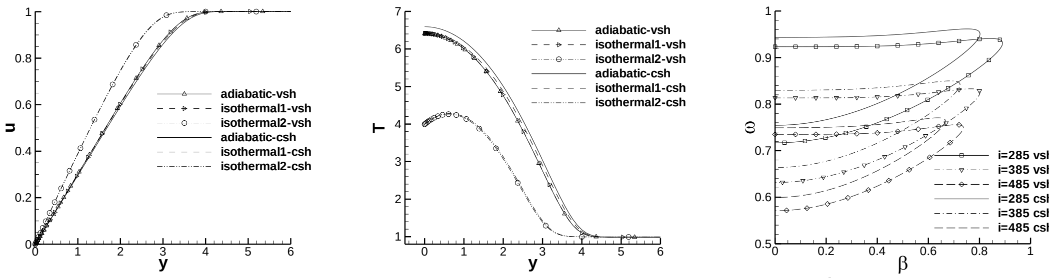

a Velocity b Temperature c 2nd mode neutral curves

Fig.1 comparison results

For the case of oncoming Mach number 8, the same trend with the case of Mach number 6 can be found. So it is not shown here. Therefore, it is reliable using the Blasius solution as the basic flow to analyze the stability of hypersonic boundary layer and predict the location of transition in the case of two-dimensional flow.

## Results

For the condition of oncoming flow Mach number 6, three cases of wall conditions are calculated: adiabatic, isothermal 1 (Tw=1606k, the wall temperature of adiabatic wall of Blasius solution), isothermal 2 (Tw=1000k). The basic flows of three cases are shown in fig.2a、b (550 dimensionless units along the flow).




a Velocity b Temperature c 2nd mode neutral curves

Fig.2 basic flow of three cases and the 2nd mode neutral curves of adiabatic wall

The 2nd mode neutral curves of different profiles along flow at adiabatic wall condition compared with perfect air are shown in fig.2c.In which i=285 expresses the 285 dimensionless units along flow; bl, results obtained from Blasius solution; vsh, acronym of variable specific heat ; csh, acronym of constant specific heat. From fig.2c, it can be seen that, compared with the perfect gas, the neutral curves of DNS have relatively large difference at the same location, the curves locating below and the unstable region expanding; the neutral curves move down and the unstable region narrows along flow. The two cases of isothermal wall have the same trend by comparing the results.

These conclusions have agreement with previous studies [5], so the variable specific should be a considered in the stability analysis of hypersonic boundary layer.

## References

[1] Tannehill J C, Mugge P H: Improved curve fits for the thermodynamic properties of equilibrium air suitable for numerical computation using time-dependent or shock-capturing methods. *NASA CR-2470*, 1974.

[2] Srinivasan S, Tannehill J C, Weilmuenster K J: Simplified curve fits for the thermodynamic properties of equilibrium air[R]. *NASA RP-1181*, 1987.

[3] Gupta R N, Yos J M,Lee K P,Thompson R A: A Review of Reaction Rates and Thermodynamic and Transport Properties for an 11-Species Air Model for Chemical and Thermal Non-equilibrium Calculations to 30000K. *NASARP-1232*, 1990.

[4] Iannelli J: Direct computation of thermodynamic properties of chemically reacting air with consideration to CFD. *Int. J. Numer. Meth. Fluids*. **43**:369-406, 2003.

[5] Wenli Jia, Wei Cao: The effects of variable specific heat on the stability of hypersonic boundary layer on a flat plate. *Appl.Math. Mech.-Engl.Ed*. **31**:34-940, 2010.

[6] Ming Yan: The stability analysis and transition estimation of the supersonic boundary layer over blunt cone at small angle of attack. *PhD Thesis*, Tianjin University, 2008.

[7] Zhihua Wu, Ge Guo, Liang Li, Zhengping Feng. Numerical method for real gas flows, *Journal of Xi'an JiaoTong University*, **40:510**-513, 2006.

[8] Gottlieb S, Shu C W, Total variation diminishing Runge-Kutta schemes. *Math. Comp.* **67**:73-85, 1998.

[9] Poinsot T J, Lele S K, Boundary conditions for direct simulations of compressible viscous flows. *Journal of Computational Physics.* **101:** 104-129, 1992.