A WEAKLY NONLINEAR MECHANISM FOR MODE SELECTION IN SWIRLING JETS

Philippe Meliga*, François Gallaire**& Jean-Marc Chomaz ***a)
*M2P2, CNRS-Universités d'Aix-Marseille, 13451 Marseilles, France
**LFMI, École Polytechnique Fédérale de Lausanne, CH1015 Lausanne, Switzerland
***LadHyX, CNRS-École Polytechnique, 91128 Palaiseau, France

Summary The spiral vortex breakdown of swirling jets is revisited using linear and nonlinear global stability analyses. We address the question of mode selection describing the bifurcations undergone by the steady axisymmetric solution. A model based on the normal form theory is presented, which predicts the existence of two distinct solutions, corresponding to pure single and double helices. The patterns and related spatio-temporal properties match well those of the solutions observed in previous direct numerical simulations, with satisfactory agreement on the bifurcation thresholds.

INTRODUCTION

The dynamics of nominally steady, axisymmetric swirling jets issuing in an infinite medium has been investigated in the frame of direct numerical simulations (DNS) by Ruith *et al.* (2003) [1]. When the swirl parameter S, which compares the intensity of the azimuthal to axial velocities, is sufficiently large, the vortex breaks down. A steady axisymmetric bubble enclosing a finite region of recirculating fluid forms first. A finite time is needed before large-scale spiral waves develop, wrapped around and behind the axisymmetric bubble. In the end, these authors report a limit cycle behaviour characterized by either a single helix structure (azimuthal wavenumber $m = -1$, moderate swirl), or a double helix structure (azimuthal wavenumber $m = -2$, large swirl). We revisit this problem using global linear and nonlinear bifurcation analyses. We address the question of mode selection computing the normal form, i.e. the system of amplitude equations preserving the problem symmetries and describing the leading-order nonlinear interactions between modes. We analyze the sequences of bifurcations predicted by the normal form, and compare the pattern and symmetries of the stable solutions to those observed in the DNS.

METHODOLOGY AND RESULTS

In the following, we use standard cylindrical coordinates r, θ and z. We consider a homogeneous, incompressible fluid whose motion is described by a state vector $\boldsymbol{q} = (\boldsymbol{u}, p)^T$ solution of the three-dimensional (3D) Navier–Stokes equations, with p the pressure and $\boldsymbol{u} = (u, v, w)^T$ the velocity field of radial, azimuthal and axial components u, v and w. Our focus is on a Grabovski & Berger vortex [2] whose non-dimensional velocities read

$$u(r) = 0\,, \qquad v(r \leq 1) = Sr(2 - r^2)\,, \qquad v(r > 1) = S/r\,, \qquad w(r) = 1\,, \tag{1}$$

using the uniform jet velocity and the jet characteristic core radius as reference scales. This leaves us with two free control parameters, namely the Reynolds number Re, and the swirl number S defined in (1) as the non-dimensional axial vorticity on the axis.

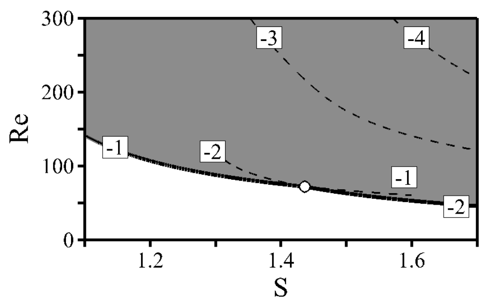

Figure 1. Boundary separating the unstable domain (grey shade) from the stable domain in the (S, Re)-plane. The thick solid line is the overall neutral curve, while the thin dashed lines are the neutral curves pertaining to azimuthal wavenumbers $m = -1$ to $m = -4$, as indicated in the superimposed labels. The symbol marks the codimension-two point $S = 1.436$, $Re = 71.95$.

The system possesses the symmetry group $SO(2)$, hence its base state is a steady axisymmetric solution which we compute using the Newton method. In the linear framework, one aims at characterizing the stability of disturbances expanded in terms of normal modes according to

$$\boldsymbol{q} = \hat{q}(r, z)\mathrm{e}^{\mathrm{i}m\theta + (\sigma + \mathrm{i}\omega)t} + \hat{q}^*(r, z)e^{-\mathrm{i}m\theta + (\sigma - \mathrm{i}\omega)t}\,. \tag{2}$$

a) Corresponding author. E-mail: chomaz@ladhyx.polytechnique.fr.

In (2), the superscript $*$ indicates complex conjugation. $\hat{q} = (\hat{u}, \hat{p})^T$ is the so-called global eigenmode of growth rate σ and pulsation ω, which we compute using a Krylov iterative method. This allows identifying several Hopf bifurcations involving time-periodic, helical modes ($\omega \neq 0$, $m \neq 0$). The boundary of the stability domain in the (S, Re)-plane is shown as the thick solid line in figure 1, the flow being unstable for combinations of parameters located in the grey shade, and stable otherwise. The neutral curves pertaining to each individual wavenumber are also shown as the superimposed thin dashed lines. We find that the transition to helical instability is led either by a $m = -1$ mode (mode A, $S < 1.436$) or a $m = -2$ mode (mode B, $S > 1.436$). Higher-order modes become subsequently unstable at larger swirl ($S > 1.36$ for $m = -3$, $S > 1.57$ for $m = -4$), consistently with previous results issuing from local stability analyses of model, parallel swirling jets, but our focus is only on the $m = -1$ and $m = -2$ modes, which are the ones bearing relevance for interpreting the DNS results. All modes of interest actually correspond to spiral perturbations, best understood in terms of pure helical waves, found to rotate in time in the same direction as the swirling flow while winding in space in the opposite direction.

We now move on to the next step and compute the weakly nonlinear dynamics near the codimension-two point $S = 1.463$, $Re = 71.95$ at which the neutral curves of modes A and B intersect. The normal form describing this double-Hopf interaction is given by

$$\mathrm{d}A/\mathrm{d}t = \alpha_A(1/Re_c - 1/Re)A + \beta_A(S_c - S)A - \mu_A A|A|^2 - \nu_A A|B|^2\,, \tag{3a}$$
$$\mathrm{d}B/\mathrm{d}t = \alpha_B(1/Re_c - 1/Re)B + \beta_B(S_c - S)B - \mu_B B|B|^2 - \nu_B B|A|^2\,, \tag{3b}$$

where A and B denote the unknown amplitudes of the bifurcating modes. We use the adjoint method to compute numerically all coefficients as the scalar product between novel sensitivity functions to axisymmetric and helical disturbances, applied to second-order mean flow corrections and harmonics. We obtain

$$\begin{array}{llll} \alpha_A = 15.0 + 12.3\mathrm{i}\,, & \beta_A = -0.282 + 0.112\mathrm{i}\,, & \mu_A = 1.20 + 0.198\mathrm{i}\,, & \nu_A = 3.06 - 0.995\mathrm{i}\,, \\ \alpha_B = 14.1 + 28.1\mathrm{i}\,, & \beta_B = -0.472 + 0.0963\mathrm{i}\,, & \mu_B = 0.836 - 1.07\mathrm{i}\,, & \nu_B = 0.553 - 0.548\mathrm{i}\,. \end{array} \tag{4}$$

The solutions to our normal form with analytically computed coefficients have been determined using the `AUTO` continuation software [3] but turn to be known analytically. Results can be synthetized as follows: at large swirl, the trivial axisymmetric solution bifurcates to the double helix (figure $2(a)$), i.e. the limit cycle for the complex Stuart–Landau equation for B, which keeps being the only stable solution. At moderate swirl, it bifurcates first to the single helix (figure $2(b)$), i.e. the limit cycle for the complex Stuart–Landau equation for A, and subsequently to the double helix through a couple of subcritical Neimark-Sacker bifurcations yielding hysteresis over a finite range of Reynolds numbers. The domains of existence of the various solutions predicted in return compare well with the DNS calculations. For instance, the DNS exhibits a bifurcation from the single to the double helix in the range $120 < Re < 150$ for the swirl value $S = 1.3$. For the same setting, the domains of existence of the single and double helices are respectively $Re < 136$ and $Re > 118$. This agreement is particularly remarkable in so far that neither the "first to bifurcate", nor the "largest growth rate" classical criteria can be invoked to explain the mode selection observed in the DNS.

(a)

(b)

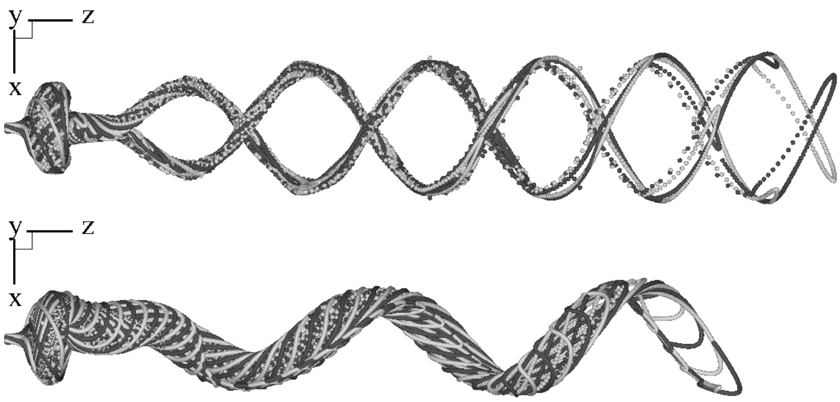

Figure 2. Visualization of the stable solutions based on numerical dye lines at $t = 70$. (a) Double helix at $Re = 150$, $S = 1.3$ based on the second-order analytic solution reconstructed from (3). (b) Same as (a) for the simple helix at $Re = 120$, $S = 1.3$.

References

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