TRANSITION DELAY OF A TWO-DIMENSIONAL CHANNEL FLOW DUE TO LONGITUDINAL WALL OSICILLATION

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Summary Stability of a two-dimensional channel flow with longitudinal wall-oscillation is investigated using the Floquet theory. To do this, time dependent Orr-Sommerfeld equation is derived from the disturbance equation by the use of the collocation method. Since the system is fundamentally linear, the velocity profiles at each instant are given from the linear superimposition of exact solution of the plane Poiseuille flow with the Stokes layer. The Reynolds number defined by the maximum speed of the flow and a half of distance between the walls is fixed to 10,000 which corresponds to the supercritical condition. Depending on the parameters of the wall oscillation, it is found that the Floquet exponent is negative for some cases. This suggests that the flow can be stabilized by the wall oscillation. The results obtained from the direct numerical simulation agree well with the results by the Floquet analysis.

INTRODUCTION

Stabilising effect of wall oscillation in the spanwise direction on two-dimensional channel flow is firstly found by Jung et.al.[1], and the subsequent study by Quadrio and Ricco[2] numerically estimated the drag reduction of about 40%. In the present study, the longitudinal wall oscillation is examined because such system can be presumed as two-dimension and thus enables to execute the analytical approach. Since this system has periodicity in time, the Floquet theory might be applied. However, two difficulties exist. Fist one is how we can obtain a periodic linear ordinary equation. The next one is how we can obtain the velocity profiles at each instant. For the latter, a model flow is proposed. This model is made by linear superimposition of the plane Poiseuille flow and the Stokes layer because both are the exact solutions of a linear equation deduced from the Navier-Stokes equation under two-dimension and the boundary-layer assumption. For the former one, a modal-type disturbance allowed time development is assumed in order to derive the time dependent Orr-Sommerfeld equation. The operator of differential with respect to wall-nomal direction is rewritten by the collocation method. Because our interest is stabilizing effect of wall oscillation, the Reynolds number is fixed to supercritical condition. Direct numerical simulation is also employed to confirm those results.

FLOQUET ANALYSIS

Model flow

Figure 1 shows the schematic view of the model flow. The parameters describing this system are the amplitude of the wall oscillation U_w and the frequency Ω. The Reynolds number Re=$U_{\max}\, h/\nu$ is fixed to 10,000, where ν is the kinematic viscocity.

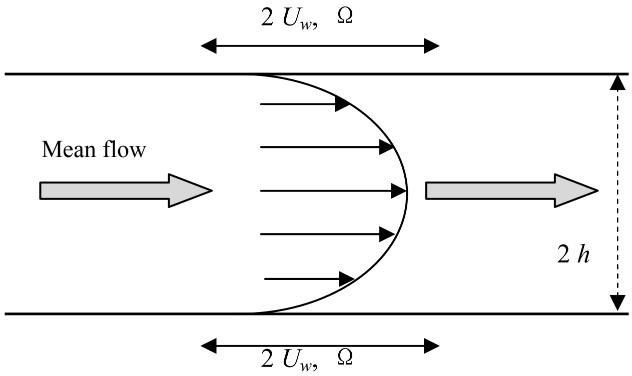

Figure 1 Schematic view of the model flow.

Time dependent Orr-Sommerfeld equation

Modal-type disturbance is assumed as the follows.

$$\boldsymbol{u}'(x,y,z,t) = \hat{\boldsymbol{u}}(y,t)\exp[i(\alpha x + \gamma z)]\,. \tag{1}$$

Here x is streamwise, y wall-normal, z spanwise direction. Then a time dependent Orr-Sommerfeld equation is derived from the disturbance equation.

$$[(\frac{\partial}{\partial t} + i\alpha U(y,t))(D^2-\alpha^2-\gamma^2) - i\alpha D^2 U(y,t)]\hat{v}(y,t) = \frac{1}{R}(D^2-\alpha^2-\gamma^2)^2\hat{v}(y,t)\,\cdot \tag{2}$$

Here, D=d/dy. In order to rewrite D to differential matrix D_{ij}, the Chebyshev spectral method is employed using the collocation points,

$$y_j = \cos\frac{\pi j}{N+1}, \quad (j=0,1,2,\cdots,N). \tag{3}$$

Then Eq.(2) can be rewritten as,

$$(D_{ij}^{(2)} - \alpha^2 - \gamma^2)\frac{d}{dt}\begin{pmatrix}\hat{v}(y_0,t)\\ \hat{v}(y_1,t)\\ \vdots \\ \hat{v}(y_N,t)\end{pmatrix} = g_{ij}\begin{pmatrix}\hat{v}(y_0,t)\\ \hat{v}(y_1,t)\\ \vdots \\ \hat{v}(y_N,t)\end{pmatrix}. \tag{4}$$

If Eq.(4) is simplified as,

$$\frac{d}{dt}F(t) = G(t)F(t), \tag{5}$$

the Floquet exponents are obtained as the follows,

$$Q = \frac{1}{T}\ln F, \tag{6}$$

here, T is the period of the wall-oscillation. To obtain F, Eq.(5) is numerically integrated during the period T, and then, the Floquet exponent can be obtained by Eq.(6).

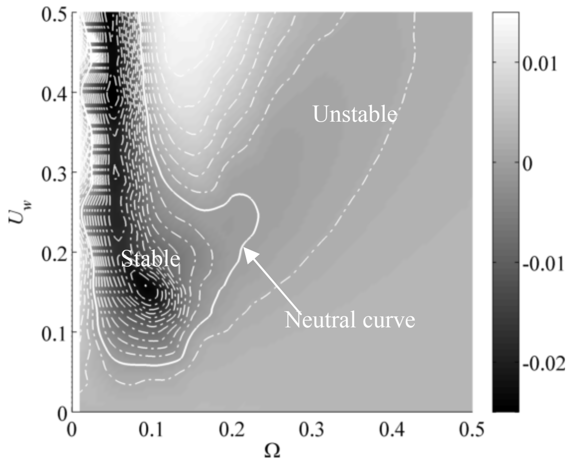

Figure 2 Contour of the Floquet exponent.

Floquet exponent

The contour of the Floquet exponent is shown in Fig.2. (0,0) point in this map corresponds to non-oscillating channel flow, and the negative region means stable. Thus, it is found that wall oscillation can stabilize even under the supercritical condition. However, since this calculation is based on the linear analysis, the accuracy might decrease at upper part of this map.

DIRECT NUMERICAL SIMULATION

Fourier spectral method

In the present study, the velocity $\boldsymbol{u}$ is expanded by the Fourier series in x direction, and the Chebyshev collocation points in y, as the follows.

$$\mathbf{u}(x, y, z, t) \equiv \sum_{\mathbf{k}} \tilde{\mathbf{u}}(\alpha, y, \gamma, t)\exp[i(\alpha x + \gamma z)] \tag{7}$$

Then, the energy norms for the Fourier modes per unit mass is defined as,

$$E(\alpha, \gamma, t) \equiv \frac{1}{4}\int_{-1}^{1}\left|\tilde{\mathbf{u}}(\alpha, y, \gamma, t)\right|^2 dy \tag{8}$$

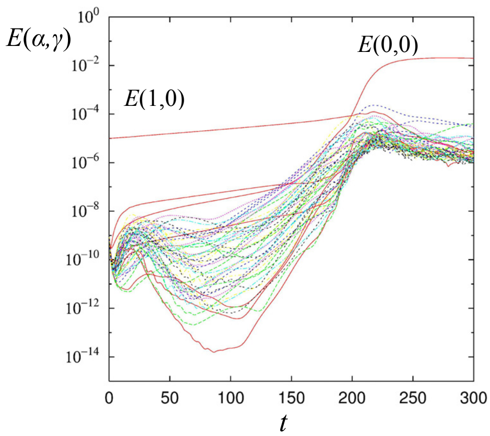

Figure 3 Energy variation of some Fourier mode for the case of (Ω,U_w)=(0,0),

Laminar-turbulent transition

Figure 3 shows time variation of the energy of some Fourier modes for the case of (Ω,U_w)=(0,0), namely non-oscillating case. The laminar-turbulent transition occurs at around t=200. On the other hand, for the case of (Ω,U_w)=(0.15,0.2), the transition delays. Also it is found that the amplification ratio of E(1,0) mode, corresponding to the Tollmien-Schlichting mode, is negative with oscillation. These results agree well with the Floquet analysis.

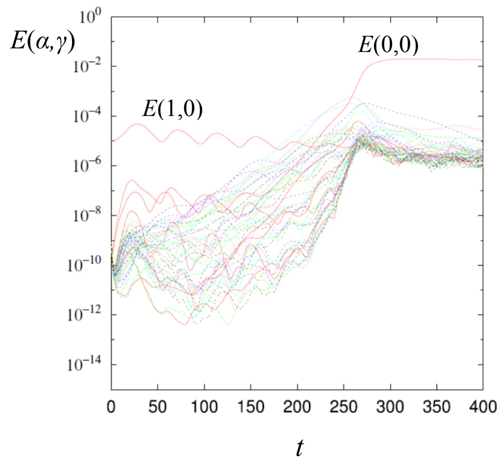

Figure 4 Energy variation of some Fourier mode for the case of (Ω,U_w)=(0.15,0.2),

CONCLUSIONS

Stability of a 2-dimensional channel flow with longitudinal wall oscillation is investigated by the Floquet theory and direct numerical simulation. A periodic linear ordinary equation for the Floquet analysis is derived from a time-dependent Orr-Sommerfeld equation and the Collocation points. Depending on the flow parameters, namely amplitude and frequency of the wall oscillation, the flow is stabilized even under the supercritical condition. Direct numerical simulation supports this feature.

References

[1] Jung, W. J., Mangiavacchi, N., and Akhavan, R.: Suppression of Turbulence in Wall-Bounded Flows by High-Frequency Spanwise Oscillations, Phys. Fluids, 1992;**A 4** (8) :1605-1607.

[2] Quadrio, M. and Ricco, P.: Critical Assessment of Turbulent Drag Reduction Through Spanwise Wall Oscillation, J. Fluid Mech., 2004;**521**:251-271.