STABILITY OF AXISYMMETRIC CONVECTION IN CYLINDERS HEATED FROM BELOW

Bofu Wang*, Dongjun Ma**& Dejun Sun*a)

*Department of Modern Mechanics, University of Science and Technology of China, Hefei 230027, China

**School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA

Summary The instability and transition of flow in a vertical cylindrical cavity with heated bottom, cooled top and insulated side wall are investigated by global instability analysis. The stability boundary are given for Prandtl numbers from 0.02 to 1.0 and for aspect ratio A (A=height/radius) equals 0.8, 0.7 respectively. For A=0.8, an "S" shape stability curve caused by hysteresis effect is found. The striking finding for A=0.7 is that very frequent change of critical mode when Prandtl number is changed, where five kinds of steady modes m=1, 2, 8, 9, 10 and three kinds of oscillatory modes m=3, 4, 6 are presented.

INTRODUCTION AND NUMERICAL METHOD

The cylindrical Rayleigh-Bénard convection has been the focus of intensive studies for a long time for its theoretical and practical importance. The flow instability determines the development of convection patterns. The instabilities of the conductive state has been well established since the work of [1, 2]. The flow structure beyond the primary threshold is independent of the Prandtl number but depends on the aspect ratio. The secondary instabilities have been investigated mainly for axisymmetric primary flow [3, 5]. The flow beyond the second bifurcation is highly nonlinear and strongly depends on both aspect ratio and Prandtl number. It is found that the stability curve is non-monotonic and there exists a linearly stable region of axisymmetric flow for certain aspect ratios and Prandtl numbers within which either a decrease or an increase of Rayleigh number will cause three dimensional flow [3]. The multiplicity of convective patterns at the same Rayleigh number was found in [4] by experiment investigation. The interesting experiment has driven further bifurcation analysis and numerical simulations [8, 9, 10]. In present research, a more detailed parameter study is performed to the second instability of the cylindrical convection. The influence of geometry and physical parameters are studied systematically by global instability analysis.

We consider an incompressible newtonian fluid confined in a vertical cylindrical cavity of aspect ratio A=H/R, where H is the height and R is the radius of the cavity. No-slip conditions are applied at all the boundaries. The top and bottom of the cylinder are assumed isothermal with a higher temperature T_h at bottom and a lower temperature T_c at top. The side wall is set adiabatic. All the physical characteristics are taken as constant except the density, which, according to the Boussinesq approximation, is taken as a linear function of temperature in the buoyancy term, ρ=$\rho_0[1-\beta(T-T_0)]$, where β is the thermal expansion coefficient and T_0 is the reference temperature (T_0=T_c). The length, time, and velocity are scaled by R, R/U_{ref}, $\nu/(PrR)$, respectively. where U_{ref} is the reference velocity, ν is the kinematic viscosity, Pr=ν/κ is the Prandtl number (κ denotes the thermal diffusivity). Nondimensional temperature is defined by θ=$(T-T_c)/(T_h-T_c)$. The dimensionless governing equations under Boussinesq approximation in cylindrical coordinates can be written as:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + Pr\,\nabla^2\mathbf{u} + (PrRa)\theta\,\mathbf{e}_z, \tag{1}$$

$$\frac{\partial \theta}{\partial t} + \mathbf{u}\cdot\nabla\theta = \nabla^2\theta, \quad \nabla\cdot\mathbf{u} = 0, \tag{2}$$

where Ra=$g\beta(T_h-T_c)R^3/(\kappa\nu)$ is the Rayleigh number and $\mathbf{e}_z$ is the unit vector in the z direction.

The time-dependent solutions of equations are obtained by using a second-order fractional-step method in three dimensional cylindrical coordinates. The global stability analysis is performed in two steps: firstly, we compute a steady solution of the nonlinear Navier-Stokes equations using Jacobian-free Newton-Krylov (JFNK) [6] method, then we determine the eigenmodes of Navier-Stokes equations linearized about this base flow using the Arnoldi algorithm from the ARPACK library [7].

RESULTS

Figure 1 gives the stability diagrams for two aspect ratios. The regions where there exists stable axisymmetric flow (AS) and where the axisymmetric flow is always unstable (AU) are divided by the stability curves. If the parameter (Pr, Ra) lies in the AS region, the flow could be axisymmetric, otherwise, the flow will be three dimensional. Figure 1(a) shows the stability diagram for A=0.8. The critical curve exhibits "S" shape. There still exits stable axisymmetric flow for 0.053$\leqslant Pr \leqslant$0.47 in certain Rayleigh number ranges. The instability modes are labeled in the figure. Similar phenomenon was found in [3] where Pr was fixed at 1 and aspect ratio varied from 0.64 to 1.1.

Figure 1(b) shows the stability diagram for A=0.7. No axisymmetric flow was found beyond the second bifurcation. The finding is the frequent change of instability mode when Pr is changed. The primary axisymmetric flow loses stability to a series of steady or unsteady three dimensional flows with the increase of Ra. It can be seen from the figure, the critical mode with relevance to the second bifurcation for small Prandtl number is steady m=2, with the increase of

a) Corresponding author. E-mail: dsun@ustc.edu.cn.

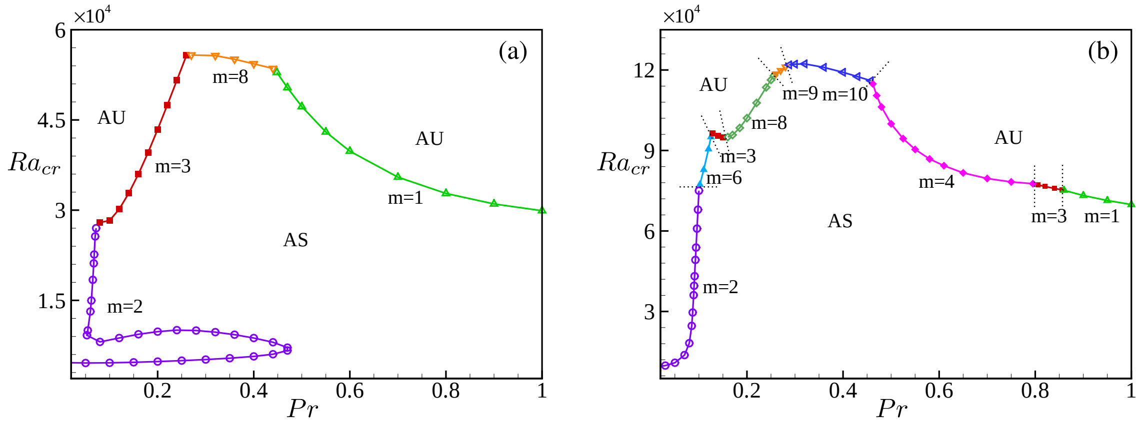

Figure 1. Stability curves for two aspect ratios: (a) A=0.8, (b) A=0.7. The curves with hollow symbols represent steady transitions and filled symbols oscillatory transitions.

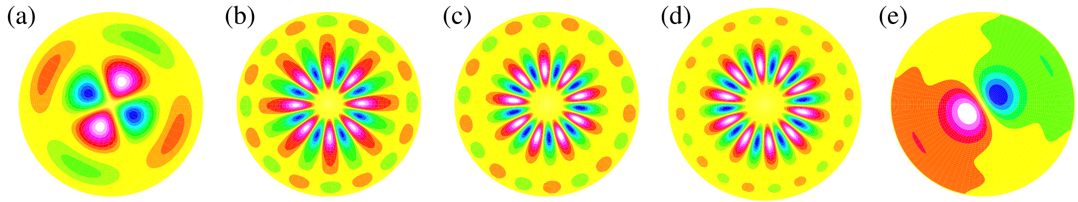

Figure 2. Critical steady modes for A=0.7: (a) Pr=0.1, Ra=75200, (b) Pr=0.2, Ra=101700 , (c) Pr=0.26, Ra=116600, (d) Pr=0.4, Ra=119000, (e) Pr=0.9, Ra=73200. Isovalues of the temperature in the horizontal midplane.

Pr, the following most unstable mode becomes oscillatory m=6, oscillatory m=3, steady m=8, steady m=9, steady m=10, oscillatory m=4, oscillatory m=3, steady m=1. The critical Rayleigh number increases with increasing Pr for 0.02$\leqslant Pr<$0.3 and grows especially rapid for 0.05$\leqslant Pr \leqslant$0.126. It decreases with further increasing Pr for 0.3$\leqslant Pr \leqslant$1. The structure of the steady modes at their thresholds is shown in figure 2 in the case of steady transitions by plots of the temperature in the horizontal midplane. It is observed that perturbation field is divided into several identical rolls. The structure of the steady state could be expected from the critical modes. The steady state is reflection symmetric in azimuthal direction and the reflection symmetry is broken by subsequent bifurcation.

Summarily, the stability diagrams for two typical aspect ratios (A=0.8, 0.7) were obtained. For A=0.8, the stability curve exhibit "S" shape due to several physical mechanisms work together. For A=0.7, the multiple modes indicate multiple flow structures triggered at the transitions. It can be seen that the instability is highly depends on Prandtl number and aspect ratio. Previous literatures have shown [3, 5] that for low Prandtl number (Pr=0.02) fluid, the flow becomes oscillatory unstable due to the inertial mechanism, and for large Prandtl number (Pr=1) fluid, the flow becomes unstable due to thermal mechanism. In a relative low cylinder, the flow tends to be oscillatory unstable, while in a tall cylinder the flow tends to be unstable through steady bifurcation. In present study, the two instability mechanism may act together. The competition and interaction of inertial mechanism and thermal mechanism lead to present result.

References

[1] Charlson GS, Sani RL. Thermoconvective instability in a bounded cylindrical fluid layer. *Int. J. Heat Mass Tran.*, 1970; 13: 1479-1496.

[2] Buell JC, Catton I. The effect of wall conduction on the stability of a fluid in a right circular cylinder heated from below. *Trans. ASME, J. Heat Transfer*, 1983; 105: 255-260.

[3] Wanschura M, Kuhlmann, HC, Rath HJ. Three-dimensional instability of axisymmetric buoyant convection in cylinders heated from below. *J. Fluid Mech.*, 1996; 326: 399-415.

[4] Hof B, Lucas PGL, Mullin T. Flow state multiplicity in convection. *Phys. Fluids*, 1999; 11: 2815–2817.

[5] Touihri R, Ben Hadid H, Henry D. On the onset of convective instabilities in cylindrical cavities heated from below. I. Pure thermal case. *Phys. Fluids*, 1999; 11: 2078-2088.

[6] Knoll DA, Keyes DE. Jacobian-free Newton-Krylov methods: a survey of approaches and applications. *J. Comput. Phys.*, 2004; 193: 357-397.

[7] Ma DJ, Sun DJ, Yin XY. A global stability analysis of the wake behind a rotating circular cylinder. *Chin. Phys. Lett.*, 2005; 22: 1964-1967.

[8] Ma DJ, Sun DJ, Yin XY. Multiplicity of steady states in cylindrical Rayleigh-Bénard convection. *Phys. Rev. E*, 2006; 74: 037302.

[9] Borońska K, Tuckerman LS. Extreme multiplicity in cylindrical Rayleigh-Bénard convection: I. Time-dependence and oscillations. *Phys. Rev. E*, 2010; 81: 036320.

[10] Borońska K, Tuckerman LS. Extreme multiplicity in cylindrical Rayleigh-Bénard convection: I. Bifurcation diagram and symmetry classification. *Phys. Rev. E*, 2010; 81: 036321.