# DNS OF TRANSITION IN CHANNEL FLOW INDUCED BY BOUNDARY LAYERS INTERACTION

Marc Buffat*[a)], Lionel Le Penven*, Julien Montagnier* & Anne Cadiou*
*Laboratoire de Mécanique des Fluides et d'Acoustique LMFA, Université de Lyon,
Université Claude-Bernard Lyon 1/ CNRS/ Ecole Centrale de Lyon/ Insa de Lyon
43, bd du 11 nov. 1918, 69622 Villeurbanne, FRANCE.

*Summary* Direct Numerical simulation of laminar-turbulent transition in channel flow induced by boundary layers interaction is presented. The transition of one of the two boundary layers is triggered by a perturbation inside the boundary layer at the entrance of the channel. This perturbation leads to a fast growth of infinitely elongated streaks, that further break down, leading to a turbulent transition of the boundary layer. The transition of the second boundary-layer is induced by the interaction with the first boundary-layer further downstream. As the configuration requires a very long channel ($150h$) at a high Reynolds number $Re_h = 20000$, the simulation requires billions of modes using spectral accurate approximations and thus the use of massively parallel computers.

## INTRODUCTION

Since the pioneer works of Orszag in 1972, the Direct Numerical Simulation (DNS) of the Navier-Stokes equation has always been a challenge for high performance computing [1]. Using massively parallel supercomputers, it is now possible to device new numerical experiments, that would be impossible a few years ago. These new accurate numerical experiments can be use for instance to get more insights in the understanding of laminar-turbulent transition of boundary layers in wall bounded flows. In the presence of free-stream or wall perturbations, turbulent transition in boundary layers can occur under subcritical conditions with respect to the linear stability theory [2]. In that case, the mechanism involving an exponentially growing mode is by-passed and transition is initiated by velocity perturbations taking the form of streaky structures elongated in the streamwise direction and modulated in the spanwise direction. Streaks are asymptotically decreasing perturbations in linear theory, but during the transient phase, they can exhibit large amplitudes capable to sustain secondary instabilities evolving to turbulence. From a mathematical point of view, the transient growth phenomenon is caused by the non-orthogonality of the Orr-Sommerfeld-Squire eigen-modes of the linear theory. If linear stability theory has improved the understanding of the first stages of the transition, direct numerical simulations are also very useful since they do not need any assumption on the perturbation amplitude and the parallelism of the base flow. Numerical study of by-pass transition in wall bounded flows is a computational challenge because the accurate description of the transition mechanisms and the turbulent development downstream need highly resolved simulations [3]. To study the transition at the entrance of a channel at high Reynolds number, we consider the development of two boundary layers on the walls of a plane channel configuration, and their interactions leading to turbulent channel flow. Direct numerical simulations of the Navier-Stokes equations are performed in such a configuration using a spectrally accurate code[4]. This configuration is elongated in the streamwise direction, so that the number of modes in the longitudinal direction is very large compared to the amount of nodes in the other directions. The simulation of such physical mechanisms typically requires billions of modes using spectral accurate approximation and thus the use of massively parallel computers.

## NUMERICAL METHOD

The numerical method is based on the decomposition of any solenoidal velocity field $\mathbf{u}$ in a plane channel into two $L_2$ orthogonal solenoidal fields as described in [4] and can be written as:

$$\mathbf{u} = \mathbf{u}_{os} + \mathbf{u}_{sq} \;\; \text{with } \nabla \mathbf{u}_{os} = \nabla \mathbf{u}_{sq} = 0 \text{ and } \int_\Omega \mathbf{u}_{os}.\mathbf{u}_{sq} d\Omega = 0$$

The $\mathbf{u}_{os}$ velocity field is defined by the wall-normal component of the velocity, $v = \mathbf{u}.\boldsymbol{e}_y$. It has zero wall-normal vorticity, its normal velocity is equal to the wall-normal velocity of $\mathbf{u}$ and the average of $\mathbf{u}_{os}$ in planes of constant $y$ coincide with the average of $\mathbf{u}$. The $\mathbf{u}_{sq}$ vector field is defined by the normal component of the vorticity, $\omega = (\boldsymbol{\nabla} \times \mathbf{u}) .\boldsymbol{e}_y$. It has zero wall-normal velocity, its wall normal vorticity is equal to the wall-normal vorticity of $\mathbf{u}$ and the average in planes of constant $y$ vanishes.

Using Fourier expansions in the $(x, z)$ planes with $MP$ modes and a Chebyshev polynomial expansion of degree $N$ in the $y$-direction, the velocity field $\mathbf{u}$ is approximated by:

$$\mathbf{u}^h(x,y,z,t) = \sum_{m=-\frac{M}{2}}^{\frac{M}{2}} \sum_{p=-\frac{P}{2}}^{\frac{P}{2}} \sum_{j=0}^{N} \left[\alpha^{mp}_{os,j}(t)\, \mathbf{u}^{mp}_{os,j}(y) + \alpha^{mp}_{sq,j}(t)\, \mathbf{u}^{mp}_{sq,j}(y)\right] \; e^{\imath(\alpha_m x + \beta_p z)}$$

The velocity fields $\{\mathbf{u}^{mp}_{os,j}(y)\}$ and $\{\mathbf{u}^{mp}_{sq,j}(y)\}$ are given divergence-free trial functions complying with the wall boundary conditions. Using a Galerkin formulation of the Navier-Stokes equations, the unknown complex spectral coefficients

a) Corresponding author. E-mail: marc.buffat@univ-lyon1.fr

$\boldsymbol{\alpha}_{os}^{mp} = \{\alpha_{os,j}^{mp}(t)\}$ and $\boldsymbol{\alpha}_{sq}^{mp} = \{\alpha_{sq,j}^{mp}(t)\}$ are solutions of $MP$ differential equations and are calculated in parallel at each time step using a Crank-Nicolson/Adams-Bashforth scheme [5].

Difficulties in the parallelization arise from the computation of non-linear terms, which require global data-transposition for the fast Fourier transforms. Using efficient 2D domain decomposition and hybrid parallel programming with MPI and high-level openMP threads [6], the NadiaSpectral code can run efficiently on massively parallel computer (IBM BlueGene/P) using 16384 cores on a $O(10^9)$ modes size simulation. On such high performance computer, typical simulation runs in 50 hours elapsed time, that would have required otherwise 100 years on a desktop computer.

## INTERACTION OF BOUNDARY LAYERS

The size of the computational domain is $150h \times 2h \times 3.2h$ where $h$ is the half-width channel size. The Reynolds number at the entrance of the channel is $Re_h = 20000$. According to previous by-pass transition study, an accurate resolution of the by-pass transitions and of the turbulent regions would require at least $17280 \times 192 \times 384$ modes ($\sim 10^9$) in order to ascertain the mesh independence of the transition scenario.

The upper laminar boundary layer is perturbed at the entrance of the channel by an optimal perturbation of small amplitude ($< 1\%$) computed from the linear Orr-Sommerfeld equations using the entrance velocity profil. This mode leads to a fast growth of infinitely elongated streaks on the upper wall side and it induces a rapid transition of the upper boundary layer (Fig. 1). The perturbation essentially acts inside the boundary layer and therefore can be experimentally generated by small obstacles with an optimal spacing on the upper wall. An additional smaller perturbation ($< 0.1\%$) is superimposed to the first one to trigger the nonlinear streaks breakdown and a turbulent transition of the upper boundary layer occurs near the entrance of the channel ($\sim 10h$). A contrario no perturbation is imposed on the lower boundary layer, which remains laminar in a large part of the channel. The turbulence developing after the by-pass transition of the upper boundary layer progressively acts on the lower boundary layer (Fig. 1a), but the transition occurs only when the channel longitudinal length is sufficiently long ($> 100h$). At $\sim 100h$, the turbulent fluctuation induced by the turbulent upper boundary layer penetrates deeply into the laminar lower boundary layer, and generate streaks. Then non linear breakdowns of these streaks induce a by pass transition of the lower boundary layer as seen on the skin friction coefficient $C_f$ on Figure 1b. Afterwards turbulence occupies all the channel width and a turbulent channel flow develops.

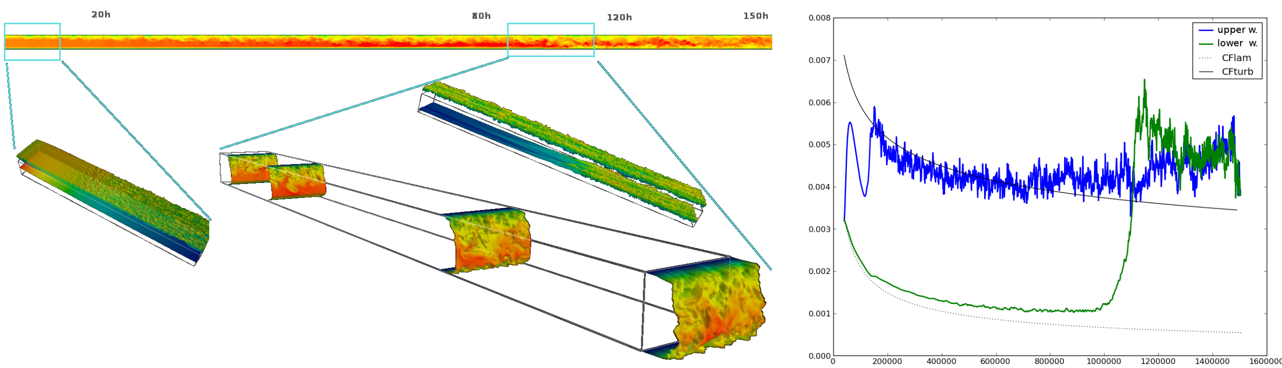


**Figure 1.** a) Instantaneous Velocity field with zoom near the channel entrance and in the interaction zone at $100h$, b) Instantaneous skin friction coefficient $C_f$ on both walls

Further statistical analysis will be presented during the conference in order to illustrate this transition scenario.


## ACKNOWLEDGMENTS

The authors would like to thank the computer center P2CHPD at Université Claude Bernard Lyon 1, member of the Fédération Lyonnaise de Modélisation et Sciences Numériques, for providing the computer facilities. This work was also performed using HPC resources from GENCI-CINES and GENCI-IDRIS (Grant 2009- [c2009026040]).


### References


[1] Moin P.& Kim J.: Tackling Turbulence with Supercomputers, *Scientific American Magazine*,2010

[2] Durbin, P.& Wu X.: *Annual Review of Fluid Mechanics*, ,**39**, 2007

[3] Schlatter P., Brandt L., de Lange H. C. & Henningson D. S: *Physics of Fluids*, ,**20**, 2008

[4] Buffat M., Le Penven L. & Cadiou A.: *Computers and Fluids* ,**42** , 2010

[5] NadiaSpectral: web site http://www.ufrmeca.univ-lyon1.fr/ buffat/NadiaSpectral

[6] Montagnier J., Cadiou A., Buffat M. & Le Penven L.: Towards Peta-scale spectral simulation for transition analysis in wall bounded flow, submitted to *Int. J. Num. Meth. in Fluids*, 2012